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Major E. H. HILLS, C.M.G., D.Sc., F.R.S., PRESIDENT, in the Chair.

Henry Frederick Baker, Sc.D., F.R.S., Lowndean Professor of Astronomy and Geometry, University, Cambridge;
Rev. Walter Bidlake, M.A., The Vicarage, Crewe;
Thomas Lydwell Eckersley, B.A., B.Sc., Khedivial Observatory, Helwân, Egypt;
Capt. William G. Walker Gilchrist, Ingleneuck, Swaythling Lawn, Southampton;
Ivor Blashka Hart, B.Sc., 9 Clapham Terrace, Leamington, Warwickshire;
Hugh Walsham, M.D., M.A., F.R.C.P., 114 Harley Street, W.; and
Charles Seymour Wright, M.A., F.R.G.S., Gonville and Caius College, Cambridge,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Jan van der Bilt, Observatory, Utrecht, Holland (proposed by J. B. Hubrecht); and
Frank Basil Cooke, Observatory, Sydney, New South Wales, Australia (proposed by W. Ernest Cooke).

Fifty-six presents were announced as having been received since the last meeting, including, amongst others :—

Photographs of the great nebula in Orion, and of Nova No. 2 Persei, etc. (4 prints and 9 lantern slides), presented by Mr. D'Esterre.

Astrographic Chart : 20 charts presented by the San Fernando Observatory.

The Constitution of Nebulæ. By J. W. Nicholson, M.A., D.Sc., Professor of Mathematics in the University of London.

It has been rendered probable, by the results of an earlier paper,* that the main chemical constituent of the planetary nebulæ, and of the others, such as Orion, with a similar spectrum, is an element, to which the name Nebulium is assigned, with a precisely indicated atomic structure. In another paper,† some other constituents, whose atomic structure is similar, have been indicated, and a relation has been obtained between the characteristics of their atoms and those of the atom of hydrogen. Moreover, it seems probable that the Wolf-Rayet stars are derived in a very simple manner, which may be traced through their spectra, from the nebulæ,‡ and it becomes essential to obtain a more complete account of the components of nebulæ, in order to proceed to a more detailed theory of the evolution of early stars. The present paper therefore takes up the further examination of the nebular spectrum, not completely, but only from the point of view of the establishment of a definite limit to the number of substances which such bodies may be supposed to contain. After the remarks in previous papers, it is not necessary to consider further, at present, the relation of such nebular substances to the ordinary terrestrial elements. Probably nebulium, for example, is not strictly an element of the type found in the Periodic Table, but must be regarded as an origin from which other elements may spring. It does not appear to possess the necessary stability. An examination of the question of stability is given in this paper, from which certain conclusions follow as to the actual physical and chemical states of a nebula. In the course of the work, some results relating to the solar corona will be used frequently, for there is reason to believe that in many important respects the solar corona is in a state analogous to that of the nebulæ.

The substances to which we have referred do not show series spectra, and have been called simple-ring systems. By this is meant, not that they differ from terrestrial substances in possessing a single ring of electrons, but that they consist of what may be

* *Monthly Notices*, Nov. 1911.
† *Ibid.*, Jan. 1914.
‡ *Ibid.*, Dec. 1913.

called a *simple* nucleus of positive charges surrounded by a ring of electrons in circular or elliptic motion. The essential difference lies in the nucleus, for there is considerable reason for the belief that the nuclei in terrestrial atoms are not simple—that is to say, they consist of a complicated system of positive and negative charges closely packed together, and not a mere positive charge. The considerations given in the paper on Wolf-Rayet stars appear to imply the supposition that the laws of series spectra are determined in some way by the structure of these nuclei, which does not, however, alter certain constants of the atom, calculated by ordinary dynamics on the assumption that the nucleus is simple. Series spectra would then be the result of an action, probably kinematical and somewhat after the manner of a series of complicated Doppler effects, of the nucleus, or perhaps of some tube of force or other agency binding the electron to the nucleus, on certain fundamental frequencies of the atom which are identical in some respects with the corresponding vibrations of simple-ring systems. On the other hand, when the nucleus is simple, this action does not take place, and the fundamental vibrations of the simple-ring atom are capable of revealing themselves as spectrum lines without modification. The success, which has attended the application of the results of calculation for simple-ring atoms to a comparison with celestial spectra, appears to show that the extent to which we must give up ordinary dynamics, for these systems at least, is very limited. A theory, such as that of Bohr,* which discards ordinary dynamics, has not been applied with success to astronomical spectra where the lines have not been actually produced in the laboratory, and, in its application to the terrestrial atoms hydrogen and helium, the main reason for its success is the approximate coincidence of a theoretical constant with Rydberg's constant of spectra. But the theory of the origin of the hydrogen atom from a primary simple-ring system, sketched in an earlier paper,† gives, as the result of calculation from the wave-lengths of measured lines in the spectra of nebulæ due to *the other* simple-ring systems, a value for the constant limit of the hydrogen spectrum of 3647·10 as against 3647·14, the experimental value. This is a much more accurate correspondence than that obtained by Bohr, and if conflict should afterwards appear between the two points of view, this accuracy must be taken into account. Points of conflict have not, however, been as yet suggested: for example, the two theories give the same constitution for the atom of hydrogen, except that the dynamical one is somewhat more specific.

If both the views are correct, as is quite possible, it would seem that one relates only to simple-ring systems, and the other only to terrestrial atoms. The fact that the second discards dynamics more completely, then, would imply that the necessity for this change in the fundamental dynamics arises when the nucleus

* *Phil. Mag.*, July 1913.
† *Monthly Notices*, Jan. 1914.

is modified and becomes endowed with a structure. Later in this paper, some investigation is given of the exact extent to which ordinary dynamics must be changed in the consideration of the astronomical atoms.

The System of Simple-ring Atoms.

The simple-ring atom with a nucleus $+e$ and one electron has been shown to give a wave-length $\lambda = 1823{\cdot}55$, half the wave-length of the limit of Balmer's formula. It must be presumed, though its wave-length is not in a part of the spectra of nebulæ which can be mapped, to be a constituent of nebulæ, for its first evolution product is hydrogen, an important constituent of nebulæ.

The simple-ring atom with a nucleus $2e$ and two electrons is present strongly in nebulæ, and was very strong in Nova Persei. Its chief line is $\lambda 3868{\cdot}88$.

The atom with nucleus $3e$ and three electrons is present less strongly in nebulæ, and its chief line is $\lambda 5187$, which is comparatively weak in all cases of its occurrence.

Nebulium is the atom with nucleus $4e$, giving, as the strongest nebular lines, $\lambda 5006{\cdot}89$ and $\lambda 4363{\cdot}5$.

Protofluorine, a main constituent of the solar corona, has a nucleus $5e$. No suggestion has yet been made of its presence in nebulæ.

These atoms, when neutral, are such that the angular momentum of any electron follows a law

$$\frac{\mathrm{E}}{\nu} = \mathrm{A} + \mathrm{B}\nu + \mathrm{C}\nu^2,$$

where E is the total angular momentum, ν the strength of the nucleus, and E/ν the angular momentum per electron. A, B, C are constants so long as the atom remains electrically neutral.

The actual values of E/ν are given by the scheme

$\nu = 1$	$\nu = 2$	$\nu = 3$	$\nu = 4$	$\nu = 5$
$\mathrm{E}/\nu = 195{\cdot}92$	$374{\cdot}04$	$500{\cdot}76$	$576{\cdot}08$	$600{\cdot}00$

on a certain scale which allows the principal wave-length to be calculated from the formula

$$\lambda = (0{\cdot}062358\mathrm{E}/\nu)^3/(\nu - \tfrac{1}{4}\mathrm{S}_n)^2.$$

These numbers rise to a maximum, and include the five simple-ring elements already, to some extent, treated in earlier papers. The formula leads further to the sequence

$\nu = 6$	$\nu = 7$	$\nu = 8$	$\nu = 9$	$\nu = 10$
$\mathrm{E}/\nu = 572{\cdot}52$	$493{\cdot}64$	$363{\cdot}36$	$181{\cdot}68$	$-51{\cdot}40$

and these must be regarded as possible atoms whose existence should be looked for, in celestial bodies showing the other five,

and especially in the nebulæ and novæ. We find that $\nu = 10$ gives E/ν negative, so that there is a break in the sequence when $\nu = 10$. As already suggested, this appears to mean that only nine simple-ring systems are possible; and if we take hydrogen, the evolution product of the first, out of the Periodic Table—a process with which chemical opinion is inclined to agree,—the remaining eight simple-ring systems are equal in number with the elements in the first two rows of the Periodic Table,—suggesting that a clue to this arrangement may ultimately be obtained from the present theory. That such a result should follow purely from the theory of nebulium, protofluorine, and another simple-ring system tends to remove the earlier theory of these elements out of the region of pure speculation altogether, if it is not already removed sufficiently by its correspondence with the spectra of hydrogen and of the Wolf-Rayet stars. It indicates that the angular momentum principle, first enunciated in connection with the coronal spectrum, may be not only the physical basis of Planck's law, but also the basis of all the possible arrangements of electrical charges into the form of ordinary matter; and the success which Bohr has obtained with it is only another illustration of its essential character to any form of theory which can now correspond with the known facts relating to atoms. Even the theory of the magneton can now be expressed entirely as a manifestation of the angular momentum principle.

Another numerical relation which may be of importance is that the value of E/ν when $\nu = 9$ is *exactly* half the value when $\nu = 8$. These are the last two members of the sequence before the break occurs, and the relation is so close that it can hardly be accidental. To one part in 20,000, in fact, it is exact.

We may, in the next place, calculate the principal wave-lengths emitted by these new simple-ring atoms ranging from $\nu = 6$ to $\nu = 10$. They are the values of $2\pi C/\omega$, where C is the velocity of light, and ω is the angular velocity in the ring. They are expressed by the formula quoted already. We find

$$6 - \tfrac{1}{4}S_6 = 4{\cdot}17265, \quad 7 - \tfrac{1}{4}S_7 = 4{\cdot}6952, \quad 8 - \tfrac{1}{4}S_8 = 5{\cdot}1951,$$
$$9 - \tfrac{1}{4}S_9 = 5{\cdot}6752;$$

and if λ_ν is the wave-length for the ring atom with nucleus ν when neutral,

$$\lambda_6 = 2613{\cdot}57 \qquad \lambda_7 = 1323{\cdot}12$$
$$\lambda_8 = 431{\cdot}023 \qquad \lambda_9 = 45{\cdot}1477$$

Thus the wave-lengths very rapidly diminish, and it is evidently impossible to obtain confirmation of the existence of the last two substances, except by extrapolation from the existence of the other members of the sequence. But the first at least, $\nu = 6$, definitely exists, and, as will be shown later, its first evolution product gives the strong ultra-violet radiation at λ 3483 recently found by Max

Wolf * in Wolf-Rayet stars. Like other calculations which have since been confirmed by recent observations on the Wolf-Rayet stars, this particular result was obtained before the appearance of Wolf's paper.

These principal vibrations in the planes of the atoms correspond to the case $q=\omega$, where $q/2\pi$ is the number of vibrations per second calculated in the ordinary way. They are necessarily accompanied by the strongest transverse vibration of class zero,—in the phraseology of earlier papers,—whose value of q is

$$q=\omega\left(\frac{\nu}{\nu-\frac{1}{4}S_n}\right)^{\frac{1}{2}}\left(1+\frac{n}{2}\frac{m}{M}\right),$$

where n is the number of electrons. For an atom with a nucleus $2e$, $m/M=0{\cdot}00166$,† and since the inertias of nuclei are proportional to the squares of their charges, when these nuclei are simple, we have the following sequence of values:—

$\nu=6$	$\nu=7$	$\nu=8$	$\nu=9$
$m/M=0{\cdot}000184$	$0{\cdot}000135$	$0{\cdot}000104$	$0{\cdot}00008$

Calculation gives the following values for the wave-lengths due to the transverse vibrations of these atoms:—

$\nu=6$	$\nu=7$	$\nu=8$	$\nu=9$
$\lambda'=2178{\cdot}34$	$1083{\cdot}11$	$347{\cdot}199$	$35{\cdot}8479$

These values are not of immediate service in proving or disproving the existence of the corresponding atoms, but will be of use later. The other transverse vibrations are of minor importance.

Vibrations in the Plane of the Atom.

When we come to the consideration of the vibrations in the planes of these atoms, we meet at once with some important questions, which are raised in an acute form by Bohr's theory, relating to the systems dealt with earlier. Bohr's theory denies that the vibrations in the plane can be determined by ordinary dynamics, with perhaps the exception of the principal one, $q=\omega$. This exception is certainly necessary, for this particular frequency has always corresponded, in simple-ring systems, to a well-defined line in the spectrum. Ordinary dynamics, in the case of simple-ring systems, for which a self-contained body of theory has now been found, must therefore be partially successful at least—though there is no consequent implication that it should be successful for terrestrial atoms with compound nuclei, such as the phenomena of radio-activity require, and perhaps other arrangements, say of æther, with mechanical significance in the atomic plane. We have seen, however, in another paper,‡ that if Bohr's theory is capable of any further development in the direction of an explanation even

* *Sitzungs. der Heid. Akad.*, 22, 1913.
† *Monthly Notices*, Jan. 1914.
‡ *Ibid.*, March 1914.

of the spectrum of helium, it is necessary to change the law of force between electrons and between two nuclei in the same atom or molecule, while at the same time preserving the law between an electron and the nucleus. It is, in fact,—although this was not proved in that paper,—necessary also to give up the idea of the universal constancy of angular momentum from atom to atom, and merely to stipulate that the angular momenta bear relations to Planck's h which depend in some way on the nature of the electronic arrangement, but at the same time do possess certain singly-infinite sets of discrete values for any given atom.

Evidently, therefore, there is an essential difference between the simple-ring systems of the nebulæ and the terrestrial atoms, if Bohr's theory of the latter is correct, for the theory of the nebular structure depends on the preservation of the laws of force between electrons as well as between electron and nucleus. Thus any failure of ordinary dynamics in the terrestrial case can never imply, of necessity, a corresponding failure in the case of the nebulæ, and it is not possible to reason from one set of conditions to the other. In fact, we must regard matter, in its nebular beginnings, as being in a different state from its ultimate form in terrestrial elements —a conclusion already emphasised from its failure to give anything but a set of apparently random spectrum lines.

At one point in the theory of the simple-ring atoms, however, Planck's constant enters—in the relation between different systems by virtue of angular momenta; and we may ask—Is this already an infringement of the ordinary dynamics, and should not energy, when radiated, be radiated in quanta? This does not appear to be necessary. For the use of electrostatic forces is only a means of mathematically representing the real binding of an electron in the atom, which may be a complicated process, for which, however, mechanisms can be imagined which would preserve the angular momentum. Bohr has objected to the application of ordinary dynamics to the vibrations of such a system because the angular momentum has to remain constant, stating that for systems whose "angular momentum is a function of the energy," any loss of energy causes a continual change of angular momentum, and a continual shift of the corresponding spectrum line. This criticism is only valid if we suppose that the energy radiated as a spectrum has its actual source in the atom. Nevertheless he is prepared to accept the existence of the coronal lines as an illustration of selective scattering, by the atom, of incident radiation, such scattering being determined by the frequencies, transverse at least, calculated by the ordinary dynamics.

We do not, however, know the energy of the atom, but only the amount radiated during its passage from one configuration to another. This is the amount of which the angular momentum is a function in atoms of the Saturnian type, and it is possible that the energy concerned in the spectrum may come from the atom without altering the frequency. But it is more reasonable to suppose that the evolved energy is never the atomic energy proper,

but, in the case of bright-line spectra, is the energy of the small vibrations about the steady configuration of the atom, derived directly from the disturbing features outside. Bright-line spectra are not shown by atoms unless they are disturbed from outside, say by collisions, and it is then a portion of the translational energy of the atoms as a whole which passes into the free vibrations of the atomic components about their steady states, and is finally radiated off during the adjustment of the atom towards its original steady state. Bohr's theory of spectra makes the energy in spectrum lines come from the inter-atomic energy. It is nevertheless possible to regard its origin as lying in the energy supplied to the atom from outside, and causing it to vibrate. There is then no difficulty in regard to the homogeneity of the emitted light, nor in regard to interference and allied problems which appear to present insuperable difficulties to Bohr's view. Regarded in this way, the so-called selective scattering is the invariable agent in the production of bright-line spectra, and it does not involve of necessity a departure from ordinary dynamics, in spite of the condition of angular momentum which determines the steady states.

It is nevertheless instructive to work out the small vibrations in the plane of the atom by another dynamics, for if the free periods of the possible vibrations exist at all,—and the vibrations presumably must exist, though not, in terrestrial atoms, sending out light of the same frequencies as their own,—Bohr's theory implies very definitely the manner in which they must be calculated. It states that the reaction or drag on the electron during a steady state in an atom, due to radiation, is zero, and implies that even in a small vibration, if such be possible, the angular momentum will not vary continuously. Since this can only change, and by a discrete multiple of $h/2\pi$, when the electron passes into another stationary state altogether, then, during a vibration of small extent about one stationary state, the angular momentum must not change at all. These are the only departures from dynamics, which is otherwise valid; and a significant illustration of the fact that there are no other departures is that, for the small vibrations transverse to the plane of the atom, Bohr finds the ordinary dynamics to be valid. These are precisely the vibrations during the course of which the angular momentum of any electron preserves its steady value.

It is therefore possible that the simple-ring atoms themselves should vibrate in their planes with unaltered angular momentum, and this possibility must be examined and compared with the astrophysical spectra. We shall find that the vibration $q = \omega$ is preserved even with this modification of ordinary dynamics.

As a preliminary, however, since the vibrations calculated by ordinary dynamics throughout have led to important lines in the coronal spectrum for the vibrations of protofluorine, we may conclude that this should be true also for the nebulæ, and we therefore calculate the vibrations of class 1 in the plane of the atom for all the simple-ring systems which may or do occur in nebulæ.

The general equation for these vibrations is, if $x = q/\omega$, to a first approximation,*

$$\left(x^2 + 3 + \frac{L_0 - L_1}{m\omega^2}\right)\left(x^2 + \frac{N_1 - N_0}{m\omega^2}\right) = \left(\frac{M_1}{m\omega^2} - 2x\right)^2,$$

where

$$L_0 - L_1 = \frac{e^2}{8a^3}\sum_1^{n-1}(1 - \cos 2r\pi/n)\left(\operatorname{cosec}\frac{r\pi}{n} + \operatorname{cosec}^3\frac{r\pi}{n}\right)$$

$$N_0 - N_1 = \frac{2e^2}{8a^3}\sum_1^{n-1}(1 - \cos 2r\pi/n)\left(\cot\frac{r\pi}{n} + \tfrac{1}{2}\tan\frac{r\pi}{n}\right)\cos^2\frac{\pi}{n}\operatorname{cosec}^2\frac{r\pi}{n}$$

$$M_1 = \frac{e^2}{8a^3}\sum_1^{n-1}\sin 2r\pi/n \cos\frac{r\pi}{n}\Big/\sin^2\frac{r\pi}{n},$$

or otherwise, if

$$S_n = \sum_1^{n-1}\operatorname{cosec}\frac{r\pi}{n}, \quad S_n' = \sum_1^{n-1}\sin\frac{r\pi}{n},$$

then

$$L_0 - L_1 = \frac{e_2}{4a^3}(S_n' + S_n)$$

$$N_0 - N_1 = \frac{e^2}{4a^3}(S_n - \tfrac{1}{2}S_n')$$

$$M_1 = \frac{e^2}{4a^3}(S_n - S_n').$$

If

$$f(x) = \left(\frac{L_1 - L_0}{m\omega^2} - x^2 - 3\right)\left(\frac{N_0 - N_1}{m\omega^2} - x^2\right) - \left(\frac{M_1}{m\omega^2} - 2x\right)^2$$

$$\gamma = \tfrac{1}{2}n\nu^2(\nu - \tfrac{1}{4}S_n)^{-2}\frac{m}{Mx^2},$$

then the second approximation is

$$q/\omega = x + \frac{\gamma}{f'(x_0)}\left\{\frac{L_0 - L_1 - 4N_0 + 4N_1 \pm 4M_1}{m\omega^2} + (3 \mp 5x)(1 \mp x)\right\},$$

the ambiguity denoting vibrations of classes ± 1 respectively, or vibrations which travel with or against the general motion of the ring.

This class of vibrations does not exist for protohydrogen. We therefore examine it, in the first place, for the neutral atom of nucleus $2e$, in which the equation of steady motion is

$$ma^3\omega^2 = \tfrac{7}{4}e^2.$$

* *Monthly Notices*, June 1912.

Here

$$S_n = 1, \qquad S_n' = 1, \qquad L_0 - L_1 = \tfrac{2}{7}m\omega^2, \qquad M_1 = 0,$$
$$N_0 - N_1 = \tfrac{1}{7}m\omega^2,$$

and the approximate equation is

$$f(x) \equiv (x^2 + 3 + \tfrac{2}{7})(x^2 - \tfrac{1}{7}) - 4x^2 = 0,$$

a quadratic in x^2. We find

$$x^2 = (3 \pm 4\sqrt{2})/7,$$

and the negative value shows that the atom is partially unstable. The real value of x becomes $x = 1{\cdot}1121$.

This vibration, being of class 1, gives approximate vibrations, for a stationary observer, according to $q = 2{\cdot}1121\omega$ or $q = 0{\cdot}1121\ \omega$. Since the principal line is at λ 4142·9, corresponding to $q = \omega$, these can never be in the observed limits of the nebular spectrum, and there is no necessity to obtain the values more accurately.

For the neutral atom with three electrons,

$$ma^3\omega^2 = e^2\left(3 - \frac{1}{\sqrt{3}}\right)$$
$$S_n = 4/\sqrt{3}, \qquad S_n' = \sqrt{3},$$

and the approximate equation is

$$\left(x^2 + 3 + \frac{7}{4(3\sqrt{3} - 1)}\right)\left(x^2 - \frac{5}{4(3\sqrt{3} - 1)}\right) = \left(2x - \frac{1}{4(3\sqrt{3} - 1)}\right)^2,$$

or on reduction

$$x^4 - 0{\cdot}880843x^2 + 0{\cdot}238314x - 1{\cdot}02146 = 0$$

with two real and two imaginary roots.

The positive real root is approximately $x = 1{\cdot}186$, and the negative root is nearly the same. They can lead, therefore, to no visible spectrum lines.

For the neutral atom of nebulium,

$$ma^3\omega^2 = e^2\left(4 - \frac{1 + 2\sqrt{2}}{4}\right)$$
$$S_n = 1 + 2\sqrt{2}, \qquad S_n' = 1 + \sqrt{2},$$

and the equation becomes

$$\left(x^2 + 3 + \frac{6{\cdot}2426}{12{\cdot}172}\right)\left(x^2 - \frac{5{\cdot}2426}{12{\cdot}172}\right) = \left(2x - \frac{1{\cdot}4142}{12{\cdot}172}\right)^2$$

or

$$x^4 - 0{\cdot}9179x^2 + 0{\cdot}4648x - 1{\cdot}5264 = 0,$$

whose real roots are $x = 1{\cdot}2391$, $x = -1{\cdot}4165$, and again neither leads to a line in the visible range of the spectrum.

Thus the neutral atoms, as far as nebulium, can give no further nebular lines in addition to those already specified in previous papers.

The case of the vibrations of the neutral atom of protofluorine has already been treated exhaustively in connection with the coronal spectrum.

Passing to the consideration of the neutral atom with six electrons, it is at present uncertain whether it will be found in the nebulæ or in the corona, for the only two wave-lengths yet calculated for it are in the extreme ultra-violet. Thus, if it is not *strongly* present, it cannot be detected theoretically at all, for the remaining vibrations, now to be calculated, are necessarily, being of class 1, considerably weaker than those already dealt with.

For this atom,

$$ma^3\omega^2 = e^2(4{\cdot}17265)$$

$$L_0 - L_1 = \frac{e^2}{4a^3}\left(7 + \frac{7}{\sqrt{3}}\right),\quad N_0 - N_1 = \frac{e^2}{4a^3}\left(8 + \frac{5}{\sqrt{3}}\right),\quad M_1 = \frac{e^2}{4a^3}\left(3 + \frac{1}{\sqrt{3}}\right)$$

and the approximate equation is

$$\left(x^2 + 3 + \frac{2{\cdot}76036}{4{\cdot}17265}\right)\left(x^2 - \frac{2{\cdot}72169}{4{\cdot}17265}\right) = \left(2x - \frac{0{\cdot}89434}{4{\cdot}17265}\right)^2$$

or

$$(x^2 + 3{\cdot}66154)(x^2 - 0{\cdot}65227) = (2x - 0{\cdot}21433)^2$$

or

$$x^4 - 0{\cdot}99073x^2 + 0{\cdot}85732x - 2{\cdot}43425 = 0.$$

The negative root is $x = -1{\cdot}58255$, which, as a vibration of class 1, gives a vibration $q/\omega = 0{\cdot}58255$ to a stationary observer. As the principal line of the atom is at λ 2613·57, this vibration comes in the visible spectrum, and must therefore be corrected for m/M to obtain an accurate value. We find

$$\gamma = 2{\cdot}477\frac{m}{M},$$

and finally

$$x = -1{\cdot}58255 - 6{\cdot}107\frac{m}{M}.$$

Now, in nebulium, $m/M = 0{\cdot}000413$, and for this nucleus the value is reduced by the factor 9, to 0·000046, so that the corrected root is

$$x = -1{\cdot}58283,$$

and the spectrum line of class 1 has a wave-length

$$\lambda = (2613{\cdot}57)/(0{\cdot}58283) = 4484.$$

This is not a nebular line yet found, and therefore either the substance is not found in nebulæ, or the line is too weak to be seen. As this is the only line by which the question can be tested, and as it is a line which, from a theoretical point of view, should be very weak, the direct test is very unsatisfactory, and we must

rely on indirect evidence. The other solution of the equation is $x = 1{\cdot}32$, which cannot, for a vibration of classes ± 1, give a line in the visible range.

The next system cannot be tested directly at all, for the same reason.

The best indirect test of the presence of such substances in the nebulæ is obtained by examining their initial evolution products, which, if previous suggestions are correct, should appear in the nebulæ themselves or in Wolf-Rayet stars. If λ_0 is the wave-length corresponding to a vibration of the simple-ring system, the evolution product first formed should give a series

$$\lambda = 2\lambda_0 \frac{n^2}{n^2 - 1},$$

and probably also

$$\lambda = \lambda_0 \frac{n^2}{n^2 - 1},$$

where n takes integral values. This has already been applied to the products of nebulium, and to the theory of Balmer's hydrogen formula—though only partially at present, for series of this type have not been shown to constitute the only series.

The leading wave-lengths of the first product of the simple-ring atom with six electrons should be given by

$$\lambda = 2613{\cdot}57 \frac{n^2}{n^2 - 1}, \qquad \lambda = 5227{\cdot}14 \frac{n^2}{n^2 - 1},$$

and we can only expect the first or second of each series, if their behaviour is analogous to that of the series from nebulium. The first wave-lengths given by the second series are $\lambda = 6969{\cdot}5, 6222{\cdot}8$, and the first is outside the limits of the known spectra of nebulæ and of Wolf-Rayet stars.

The first series gives $\lambda = 3484{\cdot}7, 3111{\cdot}4$. The second line is again outside the known spectral limits, and the first remains as the only real test of value. *It coincides almost precisely with the strong ultra-violet radiation recently found by Dr. Max Wolf in certain Wolf-Rayet stars,* the mean of Wolf's measurements being $\lambda = 3483$. The coincidence is too striking to be dismissed. Wolf has suggested that this is identical with Palmer's line λ 345 in nebulæ—which is not supposed to be more than an approximate value. As Wolf's observations are in systematic agreement, it is probable that λ 3483 is very near the true value, and there seems to be undoubted reason for the coincidence, for the present origin can account for the strength of the line. We shall conclude, therefore, that the system with six electrons is present in nebulæ, and fairly strongly, although it does not affect the spectrum, on account of the fact that all its important lines are in the extreme ultra-violet, but that in its subsequent evolution into a Wolf-Rayet substance the wave-lengths move up the spectrum into the visible region.

If Palmer's line λ 345 is the same, in spite of the difference in the measurements, we have further information as to the chronological development of the Wolf-Rayet stars, evidenced already by λ 6583, which is a line common to certain Wolf-Rayet stars and nebulæ, and therefore denoting an early stage of development. But we shall find that it is probably a different line.

It is not possible even in this manner to test the presence of the simple-ring atom with seven electrons in the nebulæ, for all the lines which can be calculated for its products, by the Wolf-Rayet formula, fall in the ultra-violet. We can therefore only deduce its presence or absence by analogy, and the question is taken up in a later section of the paper.

Non-Newtonian Dynamics.

Let us consider now the vibrations in the plane of the atom, on the supposition that they do not, as above, follow ordinary dynamics, but follow instead the different dynamics which the general constancy of angular momentum in the steady states appears to suggest. As stated, it is possible that this angular momentum preserves its steady value during the maintenance of the vibrations: the non-existence of vibrations does not appear, on any theory, to be a possibility. Constancy of angular momentum affects the equations of motion of an electron transverse to the radius vector from the nucleus, but not the equation of radial motion. By the results of another paper,* if the vibration is proportional to $e^{i\omega t}$, this becomes, for the p^{th} electron, situated at $r = a + \rho_p$, $\theta = \omega t + \phi_p$,

$$(\mathrm{A} - mq^2)\rho_p + \mathrm{A}_1\rho_{p+1} + \cdots - 2ma\omega iq\phi_p + a\mathrm{B}_1\phi_{p+1} + \cdots = 0,$$

where

$$\mathrm{A} = \mathrm{A}' - m\omega^2 - 2ve^2/a^3, \qquad \mathrm{A}' = \frac{e^2}{8a^3}\left\{ 3\Sigma \operatorname{cosec}\frac{s\pi}{n} - \Sigma \operatorname{cosec}^3\frac{s\pi}{n} \right\}$$

$$\mathrm{A}_s = \frac{e^2}{8a^3}\left(\operatorname{cosec}\frac{s\pi}{n} + \operatorname{cosec}^3\frac{s\pi}{n}\right)$$

$$\mathrm{B}_s = \frac{e^2}{8a^3}\cos\frac{s\pi}{n}\operatorname{cosec}^2\frac{s\pi}{n}.$$

For the determination of the equation of transverse motion, there are two possible points of view. We may suppose (1) that the atomic constraint is such that the electron cannot move in the atom in such a way as to *experience* transverse force, but that, apart from this necessity, ordinary dynamics is valid. Then the angular momentum is conserved during the vibration. Or (2), that even if transverse force is experienced, the angular momentum is conserved. This is a more violent departure from ordinary dynamics,

* *Monthly Notices*, June 1912.

but as it is mathematically simpler, we may consider it first. In this case, the second equation for the electron is merely

$$(a+\rho_p)^2(\omega+\phi_p)=a^2\omega,$$

or to the first order of small quantities,

$$2\omega\rho_p+ia\phi_p q=0.$$

The two equations therefore give, for the p^{th} electron, eliminating the ϕ's,

$$(\mathrm{A}-mq^2+4m\omega^2)\rho_p+\left(\mathrm{A}_1+\frac{2\iota\omega}{q}\mathrm{B}_1\right)\rho_{p+1}+\cdots-0.$$

There are a set of equations of this type, all of which can be satisfied by writing

$$\rho_2=\omega_1\rho_1,\quad \rho_3=\omega_1\rho_2,\quad \rho_4=\omega_1\rho_3,\cdots$$

where ω_1 is one of the n^{th} roots of unity. The period equation, or equation of consistency, is

$$\mathrm{A}-mq^2+4m\omega^2+\omega_1\left(\mathrm{A}_1+\frac{2\iota\omega}{q}\mathrm{B}_1\right)+\omega_1^2\left(\mathrm{A}_2+\frac{2\iota\omega}{q}\mathrm{B}_2\right)+\cdots=0.$$

The value of ω_1 selected denotes the "class" of the vibration. Thus, if $\omega_1=e^{2\iota k\pi/n}$, the class of the corresponding vibration is $\pm k$, and classes of vibrations decrease in importance as k increases.

Now, as in *Monthly Notices*, June 1912, $\mathrm{A}=-(\mathrm{L}_0+3n_a\omega^2)$, where L_0 is defined already in the present paper. Also, if $\omega_1=e^{2\iota k\pi/n}$,

$$\omega_1\mathrm{A}_1+\omega_1^2\mathrm{A}_2+\omega_1^3\mathrm{A}_3+\cdots=\mathrm{L}_k$$
$$\omega_1\mathrm{B}_1+\omega_1^2\mathrm{B}_2+\cdots=\iota\mathrm{M}_k,$$

and therefore

$$(\mathrm{A}-mq^2+4m\omega^2+\mathrm{L}_k)=-\frac{2\iota\omega}{q}(\omega_1\mathrm{B}_1\cdots)=\frac{2\omega}{q}\mathrm{M}_k$$

or

$$\mathrm{L}_0-\mathrm{L}_k+mq^2-m\omega^2=-\frac{2\omega}{q}\mathrm{M}_k.$$

For vibrations of class zero,

$$\mathrm{M}_k=0,\qquad \mathrm{L}_k-\mathrm{L}_0=0,$$

and therefore

$$q^2=\omega^2,\quad \text{or}\quad q=\pm\omega,$$

as in the ordinary dynamics.

This result is to be expected, for in vibrations of class zero, all the electrons remain equally disposed round the ring, and there is no tendency for a change of angular momentum. Thus the strong vibration $q=\omega$ should be preserved whether ordinary dynamics is used or not.

If, again,

$$S_n = \sum_{r=1}^{n-1} \operatorname{cosec} \frac{r\pi}{n}, \qquad S_n' = \sum_{1}^{n-1} \sin \frac{r\pi}{n},$$

then as before

$$L_0 - L_k = \frac{e^2}{4a^3}(S_n' + S_n) \qquad M_k = \frac{e^2}{4a^3}(S_n - S_n'),$$

$$m\omega^2 = \frac{e^2}{8a^3}(\nu - \tfrac{1}{4}S_n),$$

and if $q/\omega = x$, we obtain the cubic equation

$$x\left(x^2 - 1 + 2\frac{(S_n + S_n')}{\nu - \frac{1}{4}S_n}\right) = -\frac{4(S_n - S_n')}{(\nu - \frac{1}{4}S_n)}$$

or

$$x^3(\nu - \tfrac{1}{4}S_n) + x(\tfrac{9}{4}S_n + 2S_n' - \nu) + 4(S_n - S_n') = 0.$$

For a neutral atom with two electrons in a ring, $S_n = S_n' = 1$, and the equation gives

$$x^3(2 - \tfrac{1}{4}) + x(2 + \tfrac{1}{4}) = 0$$

with imaginary roots. In fact, stability is never secured under the present hypothesis, and the cubic has always one real root only. We may examine it in the cases of nebulium and protofluorine.

For the case of nebulium,

$$S_n = 1 + 2\sqrt{2}, \qquad S_n' = 1 + \sqrt{2},$$

and

$$x^3\left(4 - \frac{1 + 2\sqrt{2}}{4}\right) + x\left(6 + 6\sqrt{2} - 4 + \frac{1 + 2\sqrt{2}}{4}\right) + 4\sqrt{2} = 0$$

or

$$x^3(15 - 2\sqrt{2}) + x(9 + 26\sqrt{2}) + 16\sqrt{2} = 0,$$

If we solve this by Cardan's method, writing $x = -(y + z)$,

$$y^3 + z^3 = \frac{16\sqrt{2}}{15 - 2\sqrt{2}} = 1{\cdot}85903$$

$$3yz = -\frac{9 + 26\sqrt{2}}{15 - 2\sqrt{2}}, \quad \text{or} \quad y^3z^3 = -1{\cdot}96935$$

$$y^3 - z^3 = \pm\sqrt{\{(1{\cdot}85903)^2 + 7{\cdot}87740\}} = \pm 3{\cdot}36651$$

$$(y^3, z^3) = (2{\cdot}61277, \ -0{\cdot}75374)$$

$$(y, z) = (1{\cdot}37731, \ -0{\cdot}91007),$$

and the real root is

$$x = -(y + z) = -0{\cdot}46724,$$

giving, for an observer, vibrations of classes ± 1,

$$q/\omega = 1{\cdot}46724, \quad 0{\cdot}53276.$$

The corresponding wave-lengths, derived from the principal line λ 5007, are

$$\lambda = 3413, \quad 9398.$$

The second line cannot be tested, and the first, though unknown, is in a part of the spectrum in which weak lines have not been looked for. The result is therefore negative.

The real part of the imaginary roots is 0·23362, and leads to vibrations of wave-lengths

$$(5007)/(1{\cdot}2336) = 4059$$
$$(5007)/(0{\cdot}7664) = 6534.$$

Wave-lengths in the nebular spectrum nearest to these are λλ 6548, 4069, but the correction for motion of the nucleus does not bring the values nearer to these lines, and, moreover, λ 6548 is a Wolf-Rayet line and cannot be ascribed to a weak vibration of nebulium, when λ 5007 is absent.

The test is therefore fairly decisive against the present view of the vibrations, and a more decisive one may be obtained from protofluorine, for which the vibrations calculated on ordinary dynamics are important spectral lines when they fall in the visible region. In this system,

$$S_n = 5{\cdot}50553$$
$$S_n' = 3{\cdot}07768$$
$$x^3(3{\cdot}62362) + x(13{\cdot}54280) + 9{\cdot}71138 = 0$$
$$x^3 + 3{\cdot}73738x + 2{\cdot}68004 = 0.$$

The real root becomes, by Cardan's method,

$$x = -0{\cdot}64522.$$

As the principal line is at λ 3987, this value leads to no vibrations in the visible spectrum. The real part of the imaginary roots is 0·32261, giving wave-lengths

$$\lambda = 5885, \quad 3014,$$

which again cannot be tested.

But similar calculations for negatively charged atoms are conclusive. The atoms, treated by ordinary dynamics, give important lines, but by the present dynamics give none at all which have been found. As, moreover, the present dynamics does not introduce stability into the atom, we may reject it on all counts.

The second possibility is that the vibrations occur in such a way that there is no forcive tendency for a disturbance of the angular momentum of any electron—or the transverse force on an

electron is zero in all possible vibrations. Accordingly, in the previous notation, the transverse equation is, for the p^{th} electron,

$$\Sigma\rho_{p+s}B_s - aC\phi_p + a\Sigma\phi_{p+s}C_s = 0,$$

or if ω_1 is an n^{th} root of unity

$$\rho_p\Sigma\omega_1{}^sB_s - aC\phi_p + a\phi_p\Sigma\omega_1{}^sC_s = 0$$

or

$$\iota M_k\rho_p - aC\phi_p + a\phi_p N_k = 0$$

and $C = N_0$, so that

$$\phi_p = \rho_p\iota M_k/a(N_0 - N_k).$$

But by the radial equation

$$\rho_p(A - mq^2 + \Sigma\omega_1{}^sA_s) = a\phi_p(2m\omega q - \Sigma\omega_1{}^sB_s)$$

or

$$\rho_p(-L_0 - 3m\omega^2 - mq^2 + L_k) = \iota a\phi_p(2m\omega q - M_k),$$

and finally,

$$L_k - L_0 - 3m\omega^2 - mq^2 - \iota(2m\omega q - M_k)\iota M_k/(N_0 - N_k)$$

or

$$(mq^2 + 3m\omega^2 + L_0 - L_k)(N_0 - N_k) = M_k(2m\omega q - M_k)$$

is the period equation, and is a quadratic. The equation for class $k = 0$ is not included in this case. For class 1,

$$L_0 - L_k = \frac{e^2}{4a^3}(S_n + S_n'), \qquad M_k = \frac{e^2}{4a^3}(S_n - S_n')$$

$$N_0 - N_k = \frac{e^2}{4a^3}(2S_n - S_n').$$

The quadratic only has two imaginary roots. For example, if there are only two electrons, the roots are those of

$$mq^2 + 3m\omega^2 + \frac{e^2}{2a^3} = 0.$$

Accordingly, the system of displacements is thoroughly unstable. Thus neither modification of ordinary dynamics, suggested by the law of angular momentum for steady configurations, can be valid; and the fact that vibrations in the planes worked out by ordinary dynamics for these simple systems give observed spectrum lines, indicates that the vibrations do follow ordinary dynamics.

If Bohr's theory of ordinary spectra is correct, there is an essential difference between the electronic bindings in these atoms and in the terrestrial atoms, unless we are prepared to suppose that the terrestrial atoms do not hold all their electrons stably. Even if the vibrations follow ordinary dynamics, the stability of

the simple-ring systems is considerable. Schott, for example, has worked out the effect of an incident disturbance on such a system, and finds that the amplitudes corresponding to the various modes of vibration fall away very rapidly as the class of the vibration increases. It is therefore not easy to set the atom into vibration in unstable modes, and the atom might possess a high degree of permanence. This point has been emphasised before.

The final conclusion is, however, that modifications of Newtonian dynamics for the vibrations of these simple-ring systems are not permissible—for they make the instability more pronounced, and lead to no correspondences at all with observed spectrum lines. On the other hand, in the case of negatively charged protofluorine in the corona, where vibrations following the ordinary dynamics fall in the visible spectrum, they give important observed lines. The theory of simple-ring atoms must therefore proceed on the supposition that ordinary dynamics is valid for their free vibrations.

Nebular Constitution.

The systems with nuclei $2e$, $3e$, $4e$ have been recognised directly in nebulæ, their principal lines being λλ 3869, 5185, 5007. The system with nucleus $6e$ has been deduced as present in nebulæ from its evolution product giving the radiation λ 3484 of Wolf-Rayet stars. Hydrogen appears to be derived from the system of nucleus e, and as hydrogen becomes progressively stronger in certain nebulæ as λ 5007 is relatively weaker, we may suppose that the simple-ring system, with a line in the Schumann region, is also a constituent of nebulæ. If, then, nebulæ contain the nuclei $(1, 2, 3, 4, 6)e$, why not $5e$ also? The suspicion is aroused that protofluorine may also be present, although not identified in the spectrum. A common feature of the spectra of nebulæ and of the corona has not been found.

Now the strongest lines of protofluorine in the corona are:—

λλ 3987, 3387·9, 3454, 4586, 4231.

The last two correspond to positively charged atoms. Weak nebular lines have been found by Campbell at λλ 423, 459. The strongest lines of protofluorine are 3387·9, 3454, which are very like Palmer's nebular lines λλ 337, 345. There seems to be a case for further investigation. Consider, therefore, the first evolution product of the neutral atom. The transverse wave-length of this atom is 3387·9, so that the appropriate series should be

$$\lambda = (1 \text{ or } 2)(3387{\cdot}9)\left(\frac{m^2}{m^2-1}, \frac{m^2}{m^2-q^2/4\omega^2}\right).$$

As a preliminary, we indicate certain nebular lines which have been found in the planetary nebulæ NGC 6543 and NGC 6572 by

Dr. Max Wolf recently,* and which were previously unknown. These have the wave-lengths

λλ4516·7, 4499·4, 4488·5, 4430·3, 4277·9, 4238·4, 4178·5,
4166·1, 3953·0, 3946·9, 3909·9, 3851·8, 3810·7, 3784·5,

and were found on one plate in each case, with intensities 1 or 2. Those found on more than one plate are

λλ 4190·6, 4120·2, 3639·3.

Lines probably due to helium have been omitted. None of these lines has appeared in any previous list, and it is possible that they belong to a special class of nebula. The longest exposure used was a little over twelve hours, so that they are perhaps, on the other hand, general in nebulæ, but only apparent with long exposures.

The greatest error possible in these single measurements seems to be about 3 tenth metres, this being the difference between the values found for the chief nebular lines measured on four plates.

Now the series

$$\lambda = 3387{\cdot}9\frac{m^2}{m^2-1}$$

gives the wave-lengths

$m=2$	$m=3$	$m=4$
$\lambda=4517$	3811	3614

and the first two may be the lines λλ 4517, 3811 of the preceding list. Wolf's spectrum does not extend so far as 3614 into the violet. The other series of this type to be expected from the first evolution product is

$$\lambda = 3987{\cdot}1\frac{m^2}{m^2-1}$$

with members

$m=2$	$m=3$
$\lambda=5316$,	4485·5

suggesting Campbell's line λ 531 of nebulæ, and Wolf's 4488·5 in the above list. The evidence for the existence of this substance in nebulæ is very strong, but more accurate measurements of the lines in question are desirable in order to make it conclusive.

There are other nebular lines for which an accurate estimate of wave-length is very essential. For example, the positively charged atoms of the first evolution product of protofluorine should give wave-lengths

$$\lambda = 3801\frac{m^2}{m^2-1}, \qquad \lambda = 4312\frac{m^2}{m^2-1}.$$

* *Sitzungs. der Heid. Akad.*, Jahrgang 1911.

For the first series, this give lines $\lambda = 5068$, 4276, and for the second, $\lambda = 5749$.

Wolf's measures do not extend beyond λ 5000, but he finds a line at 4277·9, and λ 5749 may be the nebular line at λ 575, which has never been accurately measured, but has been prominent in Nova Aurigæ and Nova Normæ. It is significant also, in connection with novæ, that λ 4517, which should be the strongest line in this evolution product, corresponding to a neutral atom, was very prominent in Wolf's spectrum of Nova Geminorum * between March 1 and March 25, 1913, when other nebular lines, such as λ 5007, had already appeared.

The companion series for the neutral atom, for which

$$q^2/4\omega^2 = 0{\cdot}34495,$$

or

$$\lambda = 3387{\cdot}9\frac{m^2}{m^2 - 0{\cdot}34495},$$

is not readily tested, for its leading members are $\lambda = 5172$, 3523. They are both outside the limits of Wolf's spectrum, and the other nebular spectra are not exhaustive enough for comparison. The corresponding series

$$\lambda = 3987{\cdot}1\frac{m^2}{m^2 - 0{\cdot}34495}$$

gives $\lambda = 6086,\quad 4146,$

and the second would be masked by two other lines in nebulæ. The chief line in Wolf's range of the corresponding series for a charged atom

$$\lambda = 3801\frac{m^2}{m^2 - 0{\cdot}3092}$$

is $\lambda = 4119{\cdot}4$, which may be compared with Wolf's new line λ 4120; and for the doubly charged atom, with a series

$$\lambda = 4312\frac{m^2}{m^2 - 0{\cdot}2826}$$

the first line in this range is $\lambda = 4640$, where there is a nebular line in Campbell's list, not accurately measured. Wolf's measurement is λ 4641·8, the mean of 4640·3, 4641·1, 4643·8.

In every important case, therefore, evidence is obtained of the existence of this product in nebulæ, but all the lines are weak. It does not exist, therefore, to a great extent; but at the same time, so far as the measurements go at present, it appears to be capable of explaining the following weak nebular lines:—

$\lambda\lambda$ 575, 531, 464, 4517, 4488, 4278, 4120, 3811,

of which only the first three were known before the appearance of Wolf's paper. This involves the further supposition that Palmer's

* *Sitzungs. der Heid. Akad.*, 9, 1913.

nebular lines λλ 337, 345 in the ultra-violet are also identical with the coronal lines, and that the weak nebular lines λλ 459, 423 of Campbell may also be coronal lines. But, as far as can be seen, there is no evidence against this common origin, for the coronal lines which are more characteristic and not in the ultra-violet, such as λ 5303, are not part of the protofluorine spectrum, and do not appear to be due to a simple-ring system. All the simple-ring systems thus seem to be present in nebulæ, together with the first evolution products of at least two—namely, that of the atom with a nucleus + *e* (hydrogen), and that of the atom with a nucleus + 5*e*.

We may now examine the first products of each of the other simple-ring systems in turn, in order to discover how far they are also represented. The case of nebulium has already been dealt with in the paper on Wolf-Rayet stars. The first product is not found in this case, but the later ones are. Most of the lines for the first product are so far in the red that perhaps it should not be expected.

The substance with 2*e* as nucleus should give the series

$$\lambda = 3868{\cdot}9\left(\frac{m^2}{m^2-1}, \frac{m^2}{m^2-8/7}\right).$$

[This, like previous series in this paper, is the so-called subordinate series of the paper on Wolf-Rayet stars, whose wave-lengths are half those of the stronger series, in all these cases in the infra-red.]

We find leading wave-lengths in the first series,

$$\lambda = 5158, \quad 4353.$$

The first is a nova line near the nebular stage, and the second is a nebular line. This is the second origin which it may have, for it has previously appeared as a line of nebulium.

The second series gives λ = 5416·4, which might be masked by λ5412, the Pickering line, if present, and also λ = 4431·7. This may be Wolf's new line λ 4430·3, being within the limit of error, for this wave-length depends on a single measurement. These lines are weak, so that a similar series with the other possible limit, which does not appear, is perhaps hardly to be expected. At the same time, very weak lines at 4561, 4745 should be looked for.

For the first product of the substance with a nucleus 3*e*, even the lines of the secondary series are almost uniformly far in the red. The important ones should follow the series.

$$\lambda = 4657\left(\frac{m^2}{m^2-1}, \frac{m^2}{m^2-0{\cdot}3096}\right),$$

and the first in Wolf's range is λ = 4967, but it is the third member of a series which should be very weak, for the original simple-ring system has a principal line λ 5185, which is never

strong. Very weak lines at 5239, 6209, 5048 should be looked for, but all should theoretically be weaker than the new nebular lines found by Wolf, which have now received an interpretation. In no case has a stronger line failed to appear. These lines also have all been registered in late stages of various novæ.

The evolution of the system with a nucleus 6*e* apparently proceeds more slowly, for its main line after the change from a simple-ring system is, as we have seen, λ3484—a constituent of Wolf-Rayet stars but not of nebulæ. For the results of this paper do not support the suggestion that it is identical with Palmer's λ 345 of nebulæ: the theory requires both lines, and Palmer's line certainly in the nebulæ.

Summary.

It is now possible to put forward a very definite view as to the nature of nebulæ. Their average spectrum appears to show the presence of—

(1) The various simple-ring systems, with simple nuclei, showing only spectral lines which are apparently distributed at random.

(2) The first products of an evolution of some form from these systems which leads ultimately to the Wolf-Rayet stars, in which series analogous to the terrestrial spectral series may be found.

(3) Hydrogen and helium, the simplest terrestrial elements, the former at least being included in (2).

The conclusion which must be drawn is that the chemical substance of the nebulæ consists of the very beginnings of matter. The spectrum of a nebula may be described as the spectrum of chaos. Whatever may occur with terrestrial atoms, the electrons in a nebula are not held very firmly in the atoms, and a continual interchange of electrons must be taking place, with a necessary bombardment of atoms by free electrons, to which the luminosity is probably due. The physical state of the nebula must be analogous to that of a very exhausted vacuum tube of enormous extent.

Further analysis must still be carried out in order to make the theoretical spectrum complete. In the present paper, by considering only neutral atoms except in one case, the following nebular lines, with others found in novæ, have received an interpretation:—

λλ 575, 531, 464, 459, 4517, 4488, 4432, 4278, 423, 4120, 3811, 337, 345,

with the ultra-violet line at λ 3484 in Wolf-Rayet stars. The path towards a further elucidation of the nature of these stars is now more clear.

Note on Spectroscopic Binaries and the Velocity of Light. By R. S. Capon.

(*Communicated by Prof. H. H. Turner.*)

In a recent communication to the *Astrophysical Journal* (vol. xxxvii. p. 190) Michelson has shown that the velocity of light is unaffected by reflection at a moving mirror, and he has suggested as a means of determining whether the velocity is affected by motion of the source, the comparison of the velocities of light from the two limbs of the Sun. The velocity-time curves of spectroscopic binary orbits appear to offer a convenient test for any such effect. Even a small participation of the light in the motion of the star might be expected to be exhibited, on account of the rapid motion in their orbits of some of the binaries, combined with their great distance, the final effect increasing in proportion to the velocity of the source and the length of path traversed by the light.

The only theory of light-propagation which concerns itself with the velocity of the source, and which is not disproved by Michelson's experimental result quoted above, is that of Ritz, all others requiring a change of velocity at reflection at a moving mirror. The theory of Ritz supposes that the velocity of the centre of the light-wave is constant and equal to the velocity of the source at the instant of emission. He shows (*Annales de chimie et de physique*, ser. viii. 13, 1908) that such an assumption may be made the basis of an electro-magnetic theory, which leads to results identical within the limits of experimental error with those derived from the classical theory. For greater generality, it will be assumed that the velocity of the centre of the light-wave is p times the velocity of the source at the instant of emission, so that if p is unity, the results of the Ritz theory are obtained.

It is first necessary to find the change in the expression for the Doppler effect. Let y be the radial velocity of the source measured positively away from the observer, and let v_0 be the frequency of light-vibration of the source, and c the velocity of light, *i.e.* the rate of increase of the radius of a spherical light-wave.

If v' is the observed frequency on the generally accepted theory of light-propagation (which for brevity will be called the ether theory),

$$v' = \frac{v_0}{1 + \frac{y}{c}}$$

by a well-known formula.

Under our assumption the radial velocity of the centre of the light-wave is py. It is clear therefore that the wave-length of the

light is $\frac{1}{v_0}\{c+(1-p)y\}$. The frequency v'' observed is therefore given by

$$v''=\frac{c-py}{\frac{1}{v_0}\left\{c+(1-p)y\right\}}=\frac{v_0}{1+\frac{y}{c}},$$

neglecting powers of $\frac{y}{c}$ above the first.

To this degree of accuracy, therefore, the Doppler effect is unaffected by our assumption, whatever the value of p may be. (Moreover, the law of aberration is the Bradley law for all values of p. The proof of this has been placed in an appendix, so as not to interrupt the course of the argument.) On both theories the velocity deduced from the displacement of the lines of the spectrum is the true velocity at the instant of emission of the light; but while on the ether theory it is announced at an interval after emission which is the same for all parts of the orbit (neglecting the time required to cross the orbit), the present assumption makes the interval depend on the radial velocity; in fact, if D is the distance of the source, the times t_0 and t at which the velocity y is announced for the ether hypothesis and the more general form of Ritz's assumption respectively are clearly connected by

$$t=t_0+\frac{\mathrm{D}}{c^2}py, \quad . \quad . \quad . \quad . \quad (1)$$

to the order $\frac{y}{c}$.

The velocity curve deduced by any of the known methods from an orbit whose elements are given has t_0 for abscissa and y for ordinate, while the observed curve has the same ordinate, but abscissa t if the light-velocity is supposed affected by the velocity of the source in the way assumed. It is well known, however, that in all cases the observations will fit on a theoretical or (t_0, y) curve with considerable accuracy. It is therefore necessary to prove that a (t_0, y) curve can be found to fit very closely on every (t, y) curve.

A (t_0, y) curve is defined (*Ast. Nach.*, No. 3242) by the equations—

$$\left.\begin{aligned} y&=\mathrm{K}(\cos u+e\cos\varpi)\\ n(t_0-\mathrm{T})&=u-\varpi-2e\sin(u-\varpi)+\tfrac{3}{4}e^2\sin 2(u-\varpi)+\ldots\end{aligned}\right\} \quad (2)$$

where the letters have their customary signification. The corresponding equations to the observed curve are found by replacing t_0 by $t-\frac{\mathrm{D}p}{c^2}y$. Making this substitution, and writing

$$\left.\begin{aligned} e'\cos\varpi'&=e\cos\varpi\\ e'\sin\varpi'&=e\sin\varpi+\frac{n\mathrm{DK}p}{2c^2}\\ n(\mathrm{T}'-\mathrm{T})&=\varpi'-\varpi+\frac{n\mathrm{DK}p}{c^2}e\cos\varpi\end{aligned}\right\} \quad . \quad . \quad (3)$$

we find

$$\left.\begin{aligned} y &= \mathrm{K}(\cos u + e' \cos \varpi') \\ n(t - \mathrm{T}') &= u - \varpi' - 2e' \sin(u - \varpi') + \tfrac{3}{4} e'^2 \sin 2(u - \varpi') \\ &\quad + \ldots\ldots + \mathrm{S} \end{aligned}\right\} \quad . \quad (4)$$

where $\mathrm{S} = \tfrac{3}{4}\{e^2 \sin 2(u - \varpi) - e'^2 \sin 2(u - \varpi')\} + \ldots\ldots$

If e' and $\frac{n\mathrm{DK}p}{c^2}$ are small, S will be small, and the equations (3) will give the true elements e, ϖ, and T, corresponding to the observed elements e', ϖ', and T'. The effect of the series S is discussed more fully later. From the first two of these equations, we find

$$\tan \varpi' = \tan \varpi + \frac{n\mathrm{DK}p}{2c^2 e \cos \varpi} \quad . \quad . \quad . \quad (5)$$

and it is readily seen that for all values of ϖ and e the effect is to rotate the apparent periastron towards $\varpi = \frac{\pi}{2}$, *i.e.* the apparent periastra will tend to point away from the observer. Such a tendency was observed by J. Miller Barr (*Journ. of the Royal Ast. Soc. of Canada*, vol. ii., 1908, p. 70), who found that that for 23 out of 28 binaries, ϖ lay between 0° and 180°. At first sight there appears to be some evidence here in favour of our hypothesis. Before examining this evidence more closely, a brief discussion of the present position of the anomaly of the irregular distribution of the periastra might not be out of place.

Shortly after Miller Barr drew attention to the phenomenon, Ludendorff (*Ast. Nach.*, No. 4415) pointed out that the irregularity was due probably entirely to those binaries the eccentricities of whose orbits were greater than 0·5. An examination of the more abundant data now available supports this general conclusion in that the magnitude of the effect is much greater for orbits of high eccentricity, but there is nevertheless a well-marked tendency for the periastra of those orbits which possess an eccentricity less than 0·5 to distribute themselves about $\varpi = \frac{\pi}{2}$. This is most clearly exhibited by measuring the angle made by the periastron and the line $\varpi = \frac{\pi}{2}$ positive in the direction of ϖ increasing. Calling this angle ψ', and measuring ψ' in the positive direction from 0° to 180°, and in the negative direction from 0° to −180°, the mean values of $+\psi'$ and $-\psi'$ were found for the groups of binaries whose eccentricities lie in the ranges of the first column of Table I. The corresponding mean values of $+\psi'$ and $-\psi'$ are set down in columns 3 and 5, and the numbers in the means in columns 2 and 4. No binaries were included for which the orbits of both components have been calculated; for it is customary in the calculation to include as a condition that the periastra be separated

by 180°. According to the hypothesis we are testing, however, this is not the case for the apparent positions of the periastra; each periastron should be turned through a small angle towards $\varpi = \frac{\pi}{2}$.

Table I.

e'.	No. in Mean.	$+\psi'$	No. in Mean.	$-\psi'$.
0·00–0·19	13	73·5	14	87·8
0·20–0·49	12	74·0	12	74·0
0·50–1·00	3	69·7	9	53·5

In the last group, for which e is equal to and greater than 0·5, ψ' is zero for one binary. It could not therefore be included. There are really 13 binaries in this group.

The constancy of the mean values of ψ' for the two groups of smaller eccentricity seems to show that there is a real tendency for the mean deviation (as we may call $\pm\psi'$) from $\varpi = \frac{\pi}{2}$ to be less than 90°. With regard to the last group, there are perhaps too few to draw very definite conclusions, nevertheless it seems probable that the effect is really more marked for this group. Moreover, there appears to be a lack of symmetry not noticed in the first two groups, more binaries inclining to the negative side of $\psi' = 0$. In fact, this set groups itself about the line $\varpi = 40°$, and the mean deviation from this line is about 43°. Out of the 64 orbits considered, the direction of the periastron has a component *from* the Earth in 40 cases. If all directions of the periastra are really equally probable, the chance in favour of a purely accidental occurrence of as many as 40 orbits possessing this characteristic in a selection of 64 is $\frac{1}{4}\frac{1}{0}$ ("as many as 40" being taken to mean "40 or more").

All this appears in a general way in favour of our hypothesis. The probability against the observed tendency to a regular orientation of the periastra is sufficient to make an explanation desirable, and the explanation offered appears to accord well with the facts; for, as we should expect, there is a tendency for the periastra to cluster about $\varpi = \frac{\pi}{2}$ when e' is small—that is, when the application of our formulæ is legitimate, while for larger values of e' another disturbing factor seems to be entering, and this might be supposed to originate in the neglected terms assuming a sensible value. Let us next find the approximate value of p which will eliminate the orientation effect—or, in other words, supposing the actual direction of the periastra to be equally distributed, let us determine the value of p which would cause the apparent directions to exhibit the mean deviation of Table I. Omitting for the moment the last group for which $e \geqslant 0·5$, the mean deviation for the remainder is 77°·6. Replacing ϖ' in the first two of equations (3) by the angle

ψ' defined above, and ϖ by an angle ψ related to ϖ in the same way in which ψ' is related to ϖ', we find—

$$\sin(\psi - \psi') = \frac{n\mathrm{DK}p}{2c^2} \frac{\sin\psi}{e'} \quad . \quad . \quad . \quad (6)$$

This equation holds equally for negative values of the angles as for positive values, for it is easily seen from equations (3) that when $\varpi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, ϖ and ϖ' are equal, and it follows that ψ and ψ' must be negative or positive together. Now ψ defines the position of the periastron in the true orbit, and in consequence of our assumption of the equable distribution of the directions of the true periastra, the mean value of ψ is 90°. The mean value of $\psi - \psi'$ is therefore 12°·4. In Table II. the values of e' and $\frac{n\mathrm{DK}}{2c^2}$ have been tabulated for all those binaries for which a value of D —that is, of the parallax—could be found and for which e' is less than 0·5. In eight cases the parallaxes have been taken from Prof. Plummer's hypothetical parallaxes (*M.N.*, lxxii. p. 555, and lxxiii. pp. 174, 450, and *L.O.B.*, No. 212); but in the remaining six, marked with a * in the table, values determined directly were employed. In no case has a parallax less than 0″·01 been used. Owing to the omission of the smaller values, we shall find a maximum value for p, for it has been necessary to use all values of $\psi - \psi'$ to obtain a satisfactory value of $\sin(\psi - \psi')$, and clearly $\psi - \psi'$ will be greater on the average for systems whose parallax is small. In addition, we must remember that not much stress is to be laid upon individual results, owing to the large probable errors of the parallaxes. So long as conclusions are drawn only from the mean, it is probable that they will not be much in error.

Table II.

Star.	e'.	$\frac{n\mathrm{DK}}{2c^2}$.	Star.	e'.	$\frac{n\mathrm{DK}}{2c^2}$.
α Ursæ Minoris	0·13	0·20	α Carinæ	0·18	1·08
β Persei *	0·06	6·16	η Virginis	0·40	0·40
α ,, *	0·47	0·04	ζ_1 Ursæ Majoris	0·50	1·31
λ Tauri	0·07	11·4	α Coronæ Borealis	0·39	0·46
ζ ,,	0·18	0·12	θ Draconis *	0·01	1·45
σ ,, *	0·02	0·36	α Scorpii *	0·20	0·00
κ Velorum	0·19	0·33	χ Draconis *	0·42	0·01

From this table the mean value of $\frac{n\mathrm{DK}}{2c^2e'}$ is found to be 28·0. Substituting this value in (6), and putting $\psi - \psi' = 12°{\cdot}4$, and $\frac{2}{\pi}$ for the mean value of $\sin\psi$, we find as a rough maximum value of p the number 0·012. The hypothesis of Ritz, therefore, which supposes p unity, must certainly be abandoned. Moreover, even

if we were prepared to overlook the physical improbability of an increase by about 1 per cent. of the velocity of the light by the velocity of the source, there is the additional objection that, whereas ψ' should clearly be small in general for small values of e', the evidence points rather the other way. It is the large values of e' which are correlated with the small values of ψ'. The hypothesis of Stokes, which supposes that in the neighbourhood of a moving body the ether is dragged by the body so that at the surface of the body there is no relative motion of ether and matter, would appear more suitable as an explanation. The velocity of the light would have the velocity of the source superimposed on it in the neighbourhood of the source, and would gradually assume its normal velocity as its distance from the source increased. The theoretical difficulties opposed to Stokes' assumptions appear to be insurmountable, however, and have prevented the hypothetical motion of the ether from being fully defined. Any conclusions which might be drawn would therefore be purely speculative.

One point omitted above deserves consideration. Will the value of p which was found necessary to explain the anomaly in the distribution of the periastra give rise to any sensible distortion of the observed velocity curve due to the non-vanishing of the series S? We may measure the distortion by the displacement of points of the curve along the axis of t due to the sensible value of S. If instead of S, the value $\frac{1}{2\pi}$ S is used, this displacement is measured in units of a period and may be called ΔP. From Table II. the mean value of $\frac{n\mathrm{DK}p}{2c^2}$ is found to be 0·020. If we neglect the square of this small quantity, the first term of $\frac{1}{2\pi}$ S is found to be

$$\frac{3}{4\pi}\frac{n\mathrm{DK}pe'}{2c^2}\cos(2u-\varpi),$$

or, substituting the numerical values, and giving e' its maximum value 0·5 for the orbits considered, the first term is approximately $0{\cdot}0024\cos(2u-\varpi)$. The next term will be of the order e' times this, and so on. For the values of e' we are discussing, therefore, the maximum value of ΔP will not differ much from 0·0024. This is equivalent to a displacement of less than 14 minutes in a period of 4 days; or, expressed as a displacement in the ordinate y, it causes an irregularity whose maximum value for the orbits of Table II. is about 0·8 kilometre per second. It seems probable that so small an error would not be detected.

Summary and Conclusion.

In conclusion therefore we may say that any effect of the velocity of the source on the velocity of light is certainly very small; so small as definitely to disprove the Ritz emission theory; that there is some evidence in favour of a tendency for the peri-

astra to group themselves about $\varpi = \frac{\pi}{2}$; and that this phenomenon could be formally explained by the equations (2), (3), and (4), which may be interpreted in one way as a result of the influence of the motion of the source on the velocity of light, perhaps consistent with a more complex hypothesis of light-propagation such as that of Stokes.

Appendix.

The light-waves of the ordinary electro-magnetic, or, as it has been called, the ether theory, and the Ritz theory, either in its simple form or the more general form adopted in the text, are identical in their properties. The essential difference lies in the fact that whereas the centre of the ether wave is supposed fixed in the ether at the point occupied by the source at the instant of emission of the light-impulse, the centre of the Ritz wave has a uniform velocity, which is the velocity of the source at the instant of emission, or, on the more general basis, p times this velocity. Now in the ether theory the apparent direction of the source as seen by a moving observer depends upon two factors: first, the direction of the normal to the wave front when it arrives at the observer; and second, the relative velocity of the wave and the observer, which causes the light to appear to come along a line which makes an angle with the wave normal depending only on this relative velocity, instead of along the normal itself. The way in which the angle depends on the relative velocity may be deduced from the electro-magnetic equations defining the light-wave, and of course accords with observation—otherwise the electromagnetic theory would have been contradicted by experience. Hence, for the sake of brevity, it will be more convenient here to deduce the angle from a well-known observational fact—namely, that the apparent direction of the source is the true direction if source and observer have the same velocity. Let us consider what this implies according to the ether theory. In fig. 1 the plane of the paper is supposed fixed in the ether, so that if S′ is the position of the source at the instant of emission t_e, the wave expands from S′ and clearly cuts the plane in a circle with centre at this point. If w is the common velocity of source and observer relative to the ether, the source will have arrived at S, where $S'S = w(t_0 - t_e)$ when the light-impulse is observed at O, where t_0 is the time of arrival of the impulse. Since the velocity of the light-disturbance along S′O is the velocity relative to the ether—that is, *the* velocity of light, say c,—t_0 is given by $S'O = (t_0 - t_e)c$. Now OS′ is the normal to the wave, and it is a well-known fact that the source appears at S—that is, along OS. It follows that the relative motion w of the observer and the centre of the wave must cause the apparent direction to make an angle SOS′ with the normal. This is true whatever the velocity w may be. We have therefore the following rule to determine the apparent direction of the source, given the position of the centre of the wave at the time of arrival of the

disturbance, and the relative velocity of the observer and the centre of the wave:—

If O and S′ are the positions of source and observer at the time t_0, find t_e from the formula $OS' = c(t_0 - t_e)$, and draw from S′ a vector $S'S = w(t_0 - t_e)$, where w is the velocity of the observer relative to the centre of the wave. Then OS is the required direction.

The same rule will give the angle SOS′ for the Ritz hypothesis, in virtue of the identical properties of the waves, but in this case the position of S′ and the relative velocity will both be different. For example, in the simple form of the theory, when $p = 1$, and in the case where source and observer have the same constant velocity, the source clearly appears in its true direction because the centre of the wave coincides with the source, and the relative velocity becomes zero; that is, the angle SOS′ is zero. In order to apply the above rule to stellar aberration, let us take axes fixed in the Sun, or

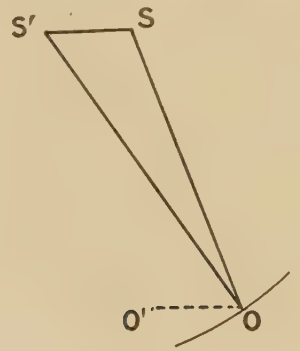

Fig. 1.

rather having such a constant velocity relative to the Sun that the elliptic motion of the Earth referred to them becomes circular motion. Let v be the constant velocity of the Earth in this motion, and let u be the velocity of the star relative to the axes. Then by our assumption the velocity of the centre of the wave relative to the axes is pu, and the velocity of the Earth relative to the centre of the wave is the resultant w of v and $-pu$. Applying the rule, we draw (fig. 2) through S′, the position of the wave at the time t_0, a vector S′S″ of length $w(t_0 - t_e)$, and the apparent direction is OS″. Now clearly we may find S″ by drawing the vector S′S‴ equal to $-pu(t_0 - t_e)$, and a vector from S‴ equal to $v(t_0 - t_e)$. In virtue of the equivalence of w and the velocities v and $-pu$, the end of this last vector will coincide with S″. But by our rule, OS‴ is the apparent direction of the star as seen from an observer attached to the moving axes, and clearly the direction OS′ is obtained from the direction OS‴ by the customary construction for determining the apparent direction of the star as seen from the Earth from the apparent direction as seen from the chosen axes. [There is a small discrepancy, which arises in the electro-

magnetic theory as in the theory we are considering. The construction of the text-books requires a length OP equal to C, the velocity of light to be taken along the apparent direction for an observer fixed to the axes, and from P a line PQ in the direction of, and equal to, v. Then OQ is the apparent direction for an observer on the Earth. This is only correct for either theory if the velocity of the light-disturbance along OQ is c, and this is not the case unless pu is zero for the one assumption, or the velocity of the ether relative to the axes is zero for the other. The difference is only of the second order, however.] We see therefore that the aberrational displacements due to the motion of the Earth in its orbit are the same on the ether and on the Ritz hypotheses. The absolute displacements are different; that is to say, the centres of the ellipses in which the star would appear to move according as one or the other mode of light-propagation

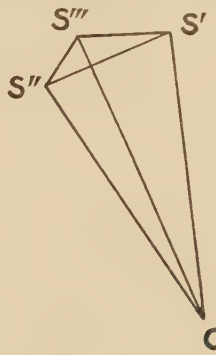

FIG. 2.

were true, would occupy different positions; but this of course affords no criterion for distinguishing between them.

If the source is not a point, small effects different from those to be expected from the classical theory of light-propagation arise. The most remarkable occurs when there is rapid rotation of a star combined with a velocity across the line of sight. If U is the velocity referred to the centre of a star of a point on the equator, and V is the velocity across the line of sight, the star is drawn out into a line of light of length $\frac{2p\mathrm{UV}}{c^2}$ radians. Giving U a value twenty times the equatorial velocity of the Sun, and putting V = 100 km. per sec., the length of the line is only $0''{\cdot}017p$. The other effects are even less noticeable.

It appears rather surprising that neither the Doppler effect nor the phenomena of aberration should betray so fundamental a change in the mode of propagation of a light-impulse as that assumed.

University Observatory, Oxford:
1914 *April* 20.

[*Note added April* 20. — After the above note was communicated, I met with a group of papers discussing the same subject. It may be convenient briefly to review the conclusions of their authors. All the discussions—at least so far as the analysis is concerned—are confined to orbits whose true eccentricity is so small that the motion may be considered uniform in a circle. On this basis, de Sitter (*Konink. Akad. Wetensch. Amsterdam, Proc.*, xvi. pp. 395–396, 1913 November 27) showed that the constant p must be less than 0·002, if the true eccentricity of β Aurigæ may be supposed zero; for the observed eccentricity is only 0·015. Professor Korteweg has pointed out, however, that there is no reason to suppose that β Aurigæ has not a small true eccentricity sufficient to cancel the apparent eccentricity introduced by the hypothesis that the velocity of light is affected by the motion of the source. Of course, if p is large, the argument holds good, for then it would be necessary to assume a large eccentricity not only for this but for the majority of spectroscopic binaries, since the majority of the observed eccentricities are small. But it appears from equations (3) of this note that, if p is small

$$e' = e + \frac{n\mathrm{DK}p}{2c^2} \sin \varpi,$$

and therefore the hypothesis for a value of p such as we have deduced to explain the distribution of the periastra, viz. 0·012, would only give rise to a slight *redistribution* of the eccentricities, for the mean value of $\frac{n\mathrm{DK}p}{2c^2} \sin \varpi$ is less than 0·020. The following communications are also due to de Sitter: *Physikalische Zeitschrift*, xiv. pp. 429 and 1267; *Konink. Akad. Wetensch. Amsterdam, Proc.*, xv. pp. 1297–1298.

P. Guthnick (*Ast. Nach.*, No. 4760, June 1913) and E. Freundlich (*Physik. Zeitschr.*, xiv. p. 835) have pointed out that a value of p different from zero would explain the distribution of the periastra. Freundlich has also suggested that the distribution may be real, the periastra tending to point away from the Milky Way. Such a supposition would give rise to the observed effect, without involving the improbability of the assumption that the orientation of the periastra is influenced by the solar system. Nevertheless, it must be remembered that for every periastron there is another periastron, namely, that of the other component of the double, separated from the first by 180°. It is clear therefore that the assumption involved is that the periastron of the *brighter* of two components of a double points away from the Milky Way. Even if we correlate brightness with mass, it appears difficult to conceive of a causal explanation of such a state of things. Moreover, I had tested this hypothesis by correlating the angle called the divergence above (the angle ψ') with the angular distance from the pole of the Milky Way. The coefficient of correlation was only 0·16, and an inspection of the

data shows that this is a measure of the opposite effect to that required. In so far as there is any association of the apparent directions of the periastra and galactic latitude, it is the binaries near the plane of the Milky Way rather than those in the higher latitudes whose periastra are directed from the earth. This explanation therefore fails completely.

I regret that my late discovery of these papers has compelled me to pass over many points of interest which arise from them. It is hoped that a further communication may be made later, in which they will be more fully discussed.]

Micrometrical Measures of 360 *Wide Double Stars.*

By W. S. Franks.

The following measures include: I. All the stars in Otto Struve's Catalogue of 256 Doubles between 32″ and 2′ distance; except such as are duplicates, and elsewhere recorded; II. The 60 objects in W. Struve's Appendix I.; III. A selection of wide Struvian pairs affected with proper motion, taken from Lewis's Memoir (lvi., *R.A.S.*) on the "Mensuræ Micrometricæ"; including also such of Σ's Appendix II. that could be readily measured with bright-field illumination; IV. Miscellaneous wide pairs, mostly from Herschel. Most of the foregoing objects do not seem to have been systematically measured since Dembowski's time, and hence it appeared desirable to repeat them. The results indicate that about one-third of the OΣΣ and Σ Appendix I. stars are affected with proper motion.

The instrument used was the 6½-inch Cooke refractor, of 82 inches focus, at Mr. F. J. Hanbury's Observatory, Brockhurst, East Grinstead, Lat. N. 51° 7′ 27″: Long. E. $0^h\ 0^m\ 2^s{\cdot}27$. The mounting is an English equatorial with polar axis of mahogany, tapered towards each end; circles 20 inches diameter, reading to 10″ of arc and 2^s of time respectively. The driving clock is of somewhat antiquated pattern, and rather irregular in action. The position micrometer is also by Cooke, but unfortunately without any provision for bright-wire illumination—therefore only bright-field illumination could be employed. A power of 235 diameters was used throughout, that being the only eyepiece with the micrometer.

The value of one revolution of the screw, as determined by a number of transits of β Ursæ Minoris, was 25″·40; this was afterwards checked by repeated measures of several wide pairs from Σ's Appendix I., the mean of which gave 25″·378 as the nearest

to accuracy—25″·38 was adopted as the final value. As a rule four measures of distance (two double-distances) were made, and three of angle—with such wide objects this was found sufficient,—the wires being always viewed either parallel to or at right angles to the line of the eyes.

The R.A. and Declination is merely approximate; the places of the OΣΣ stars being derived from vol. xl. of the *Memoirs*, and brought up to epoch 1900 from the data there given. No difficulty was experienced in finding any of the objects from these co-ordinates. The abbreviations A, D, <, >, indicate respectively angle, distance, increasing, decreasing; 2 *n*, etc. = No. of nights of observation.

I. *Otto Struve's Catalogue of* 256 *Doubles between* 32″ *and* 2′ *Distance.* (Duplicates excluded.)

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
1	0 8·4	+75 27	6·7, 7·0	102·9	75·38	1913·791	Mean 102°·8 : 75″·28 : 2 *n*. Dist. >.
				102·7	75·18	·819	
2	,, 26·1	33 2	5·5, 8·5	84·4	57·42	·791	Mean 84°·2 : 56″·56 : 2 *n*.
				84·0	55·71	·794	
3	,, 30·1	84 11	7·0, 7·5	59·9	40·04	·791	,, 60°·25 : 39″·88 : 2 *n*.
				60·6	39·72	·819	
4	,, 31·5	33 10	4·1, 8·0	173·3	35·72	·794	π Andromedæ.
5	,, 33·0	76 18	6·0, 8·0	143·5	117·06	·791	Mean 143°·2 : 116″·49 : 2 *n*.
				142·9	115·92	·819	
7	,, 41·7	50 33	7·0, 7·2	76·3	52·28	·824	,, 76°·35 : 51″·96 : 2 *n*. Dist. <.
				76·4	51·65	·832	
8	,, 42·4	12 21	8·2, 8·5	124·8	44·41	·794	
9	,, 44·3	29 54	7·0, 8·0	237·7	99·74	·824	Mean 237°·7 : 99″·10 : 2 *n*. Ang. and dist. <.
				237·7	98·47	·832	
11	1 1·6	38 6	7·2, 8·2	159·9	60·59	·832	Mean 159°·9 : 60″·75 : 2 *n*. Ang. < ; dist. >.
				159·9	60·91	·860	
13	,, 8·9	73 30	6·5, 7·2	203·5	61·42	·860	
15	,, 14·4	64 8	6·5, 8·5	348·2	53·36	·860	35 Cass. 347°·9 : 53″·21 : 2 *n*.
				347·6	53·07	·868	
16	,, 17·3	16 40	7·0, 9·0	137·7	64·02	·868	
17	,, 18·7	38 30	7·5, 9·0, 8·0, 9·0	102·3	37·31	AB ·868	Quad. 102°·1 : 37″·34 : 2 *n*.
				101·9	37·37	·879	
,,	,, ,,	,,	,, ,,	339·1	145·17	AC ·868	,, 339°·0 : 144″·66 : 2 *n*.
				338·9	144·16	·879	
,,	,, ,,	,,	,, ,,	289·4	56·98	CD ·868	,, 289°·15 : 56″·03 : 2 *n*.
				288·9	55·08	·879	

I. *Otto Struve's Catalogue of 256 Doubles between 32″ and 2′ Distance*—continued.

0ΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
19	1 23·1	+7 27	6·0, 7·5	99·1	68·40	1913·879	P. I 85–7. 99°·95 : 68″·40 : 2 *n.*
				98·8	68·40	·890	Dist. >.
20	,, 32·1	22 3	7·5, 8·5	313·8	94·48	·890	314°·0 : 94″·42 : 2 *n.* Dist. >.
				314·2	94·36	·895	
21	,, 52·4	23 7	4·7, 6·7	46·8	37·62	·890	λ Ari. 46°·4 : 37″·84 : 2 *n.*
				46·0	38·07	·895	
22	2 1·2	79 13	6·0, 7·0	275·1	55·90	·890	
23	,, 3·7	25 28	4·9, 7·7	278·1	106·40	·890	14 Arietis.
24	,, 5·8	56 45	7·0, 8·0	331·1	65·54	·895	331°·3 : 66″·12 : 2 *n.* Rapid < D,
				331·5	66·69	·903	if not an error of 10″ by De ?
25	,, 9·9	56 36	6·0, 7·0	204·0	103·30	·895	P. II 21–2.
26	,, 12·5	59 35	6·1, 6·6	199·6	63·51	·895	
27	,, 21·4	10 7	7·0, 8·0	31·1	74·19	·890	P. II 85.
28	,, 31·1	62 10	7·5, 7·9	147·6	66·75	·895	147°·6 : 67″·51 : 2 *n.*
				147·6	68·27	·903	
30	,, 33·6	8 28	7·5, 9·0	214·1	68·08	·890	213°·7 : 68″·62 : 2 *n.*
				213·3	69·16	·895	
31	,, 53·0	59 17	7·8, 8·1	228·9	73·73	·895	
33	3 13·5	62 24	7·0, 9·0, 7·2	118·5	19·98	AB ·903	Triple. AB = Σ 373 *rej.*
,,	,, ,,	,,	,, ,,	110·7	117·51	AC ·903	
36	,, 31·2	63 34	6·3, 7·3	70·8	45·62	·906	
37	,, 31·4	44 28	6·2, 6·5	95·1	41·49	·906	
38	,, 38·7	27 34	6·2, 6·3	41·9	125·64	·906	42°·05: 126″·11: 2 *n.* A and D <.
				42·2	126·58	·912	
39	,, 41·3	56 49	5·9, 6·6	75·6	57·80	·906	75°·15 : 58″·43 : 2 *n.*
				74·7	59·06	·912	
40	,, 43·4	24 4	6·5, 7·0	308·2	86·29	·906	P. III 164. 308°·3 : 86″·64 : 2 *n.*
				308·4	86·99	·923	
41	,, 49·1	4 53	7·5, 8·5	356·3	59·13	·906	
44	4 10·1	45 58	6·5, 7·0	321·4	58·30	·923	
45	,, 10·2	5 58	6·0, 6·5	315·3	65·41	·923	P. IV 24–5.
46	,, 12·9	55 18	7·0, 7·3	159·4	98·91	·939	
47	,, 12·9	50 1	6·5, 7·2	327·4	73·22	·939	327°·3 : 73″·47 : 2 *n.*
				327·2	73·72	·950	
48	,, 14·2	27 7	5·0, 8·0	249·3	53·43	·939	φ Tauri. 249°·05 : 52″·92 : 2 *n.*
				248·8	52·41	·950	A < ; D >.

I. *Otto Struve's Catalogue of 256 Doubles between 32″ and 2′ Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
49	4 13·8	+1 30	7·0, 7·0	144·8	103·74	1913·923	144°·75 : 103″·68 : 2 *n*.
				144·7	103·62	·950	
50	,, 26·4	42 51	5·2, 6·2	197·9	116·18	·961	57 Persei. 198°·35 : 116″·21 : 2 *n*. D <.
				198·8	116·24	·966	
51	,, 28·8	47 10	7·1, 7·5	198·7	58·49	·961	198°·65 : 58″·465 : 2 *n*.
				198·6	58·44	·966	
52	,, 30·2	9 57	4·0, 7·5	298·6	69·54	·950	88 Tauri. 298°·7 : 69″·72 : 2 *n*.
				298·8	69·90	·955	
53	,, 32·2	0 21	7·0, 7·2	172·7	78·43	·923	
54	,, 36·2	22 46	5·0, 7·2	213·2	62·12	·961	τ Tauri. 213°·15 : 62″·53 : 2 *n*.
				213·1	62·94	·966	
55	,, 43·7	5 3	8·0, 9·0	14·6	38·52	·923	13°·9 : 37″·82 : 2 *n*. A >?
				13·2	37·12	·950	
57	,, 54·5	60 18	4·0, 7·2	207·8	82·10	·961	β Cam. 208°·1 : 82″·20 : 2 *n*. D <.
				208·4	82·30	·966	
58	,, 53·3	14 24	5·2, 6·7, 8·0	304·8	40·61	AB ·950	304°·65 : 40″·23 : 2 *n*. P. IV 255–7. Triple.
				304·5	39·85	·955	
				...	...	·950	Stopped by clouds.
,,	,, ,,	,,	,, ,,	88·6	54·95	AC ·955	
61	5 3·2	29 40	6·5, 8·0	244·3	69·22	·977	
62	,, 6·4	6 44	7·5, 7·8	49·3	124·68	·977	49°·95 : 124″·96 : 2 *n*. A and D <.
				50·6	125·25	·986	
63	,, 23·6	39 44	6·0, 7·0	274·2	75·69	·977	
64	,, 24·0	18 20	7·2, 7·7	20·6	53·81	·977	21°·0 : 53″·33 : 2 *n*.
				21·4	52·85	·986	
65	,, 32·6	0 56	est. 8·0, 8·2 8·2, 8·4	31·1	79·00	·977	31°·5 : 78″·75 : 2 *n*. No mags given. D >?
				31·9	78·50	·986	
66	,, 41·9	24 39	6·5, 7·0	165·5	94·16	·988	P. V 214.
68	,, 49·0	13 50	7·5, 8·0	130·8	46·74	·988	131°·0 : 46″·21 : 2 *n*.
				131·2	45·68	·994	
69	6 5·4	66 10	6·7, 8·3	127·4	70·37	1914·057	127°·3 : 69″·70 : 2 *n*. A <?
				127·2	69·03	·060	
70	,, 10·0	24 0	7·0, 7·5	177·4	117·45	·057	P. VI 13–14. 177°·8 : 117″·165 : 2 *n*
				178·2	116·88	·060	
71	,, 8·8	11 50	6·5, 7·0	310·0	92·00	·011	310°·15 : 91″·465 : 2 *n*. D <.
				310·3	90·93	·014	

I. *Otto Struve's Catalogue of 256 Doubles between 32″ and 2′ Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
72	6 15·5	+59 45	7·0, 11·0, 7·5	B too small to measure		1914	Triple.
,,	,, ,,	,,	,, ,,	321·7	134·76	AC ·057	
73	,, 13·8	13 30	6·5, 7·0	44·4	72·97	·011	
74	,, 14·4	25 15	7·0, 8·5	264·8	58·95	·057	264°·4 : 59″·01 : 2 *n* D$_3$ < ?
				264·0	59·07	·060	
75	,, 14·6	18 6	7·0, 8·0	127·4	46·22	·057	127°·15 : 46″·615 : 2 *n*. D > ?
				126·9	47·01	·060	
77	,, 23·1	20 17	4·2, 8·0	329·3	112·43	·057	*ν* Gem.
78	,, 39·5	43 40	5·5, 8·0	24·4	43·78	·085	56 Aur. 24°·75 : 43″·365 : 2 *n*.
				25·1	42·95	·088	A < ; D >.
79	,, 48·7	6 50	7·0, 7·5	89·0	115·80	·060	
80	,, 52·5	14 23	7·0, 7·2, 8·0	53·4	123·85	AB ·085	Triple.
,,	,, ,,	,,	,, ,,	112·2	78·80	AC ·085	
81	,, 58·2	20 43	4·0, 7·2	351·3	96·12	·085	ζ Gem. 351°·4 : 96″·12 : 2 *n*. D <.
				351·5	96·12	·088	
82	,, 59·2	1 39	6·0, 7·0	318·6	91·37	·088	318°·35 : 91″·45 : 2 *n*. D < ?
				318·1	91·53	·090	
83	7 3·4	25 55	6·0, 7·0	84·8	111·35	·093	84°·85 : 111″·36 : 2 *n*. A > ; D <.
				84·9	111·37	·096	
84	,, 17·1	56 46	7·0, 7·2	324·8	114·84	·090	P. VII 61–2. 324°·9 : 114″·40 : 2 *n*.
				325·0	113·96	·093	
85	,, 22·5	24 52	7·5, 8·0	25·8	58·16	·093	25°·95 : 58″·015 : 2 *n*. A > ; D <.
				26·1	57·87	·096	
86	,, 23·6	14 33	7·0, 8·0	349·4	55·90	·093	
87	,, 31·7	42 43	7·0, 7·0	178·9	63·51	·123	178°·85 : 64″·02 : 2 *n*. D >.
				178·8	64·53	·131	
88	,, 43·7	0 46	7·5, 8·0	5·3	59·76	·090	5°·15 : 59″·16 : 2 *n*. D <.
				5·0	58·56	·093	
89	,, 44·5	31 52	6·5, 7·0	83·2	77·73	·123	82°·9 : 77″·825 : 2 *n*. D <.
				82·6	77·92	·131	
90	,, 53·6	63 22	6·0, 7·0	82·2	49·68	·123	82°·0 : 49″·68 : 2 *n*. D <.
				81·8	49·68	·131	
91	8 13·9	35 23	6·5, 7·5	222·2	94·35	·123	222°·05 : 94″·54 : 2 *n*. A > ; D <.
				221·9	94·73	·131	
92	,, 15·9	57 45	7·0, 8·5	181·1	58·62	·123	181°·0 : 58″·55 : 2 *n*.
				180·9	57·48	·131	

I. *Otto Struve's Catalogue of* 256 *Doubles between* 32″ *and* 2′ *Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
93	8 18·0	+42 20	6·0, 8·0	170·4	77·09	1914·123	170°·65 : 77″·41 : 2 *n*. A <.
				170·9	77·73	·131	
94	,, 30·5	14 8	7·0, 8·0	132·7	43·72	·131	
95	,, 34·7	19 54	6·3, 7·0, 7·2	61·5	92·95	AB ·140	ε Cancri. Triple.
,,	,, ,,	,,	,, ,,	87·8	99·93	AC ·140	
96	,, 45·9	26 7	7·0, 7·5, 10·5	311·4	46·31	AB ·142	312°·3 : 45″·87 : 2 *n*. D <. Trip. C too faint.
				313·2	45·43	·145	
97	9 2·5	27 57	7·5, 7·7	57·6	51·65	·142	
98	,, 11·8	7 41	7·7, 8·0	168·3	109·77	·145	168°·8 : 111″·96 : 3 *n*.
				169·0	114·06	·148	
				169·1	112·05	·153	
99	,, 22·1	46 2	5·6, 7·8	161·8	77·07	·151	161°·85 : 77″·55 : 2 *n*. 41 Lyncis. D >.
				161·9	78·03	·153	
100	,, 24·7	34 5	6·0, 9·0	129·9	63·13	·148	7 Leo Min.
101	,, 26·6	10 10	5·0, 9·5	75·0	37·12	·153	6 Leonis.
102	,, 29·8	14 32	7·7, 8·5	44·4	48·79	·153	44°·1 : 48″·725 : 2 *n*. A < ; D >.
				43·8	48·66	·156	
103	,, 45·3	19 48	8·5, 9·0	125·1	78·04	·153	125°·55 : 78″·14 : 2 *n*. A <.
				126·0	78·24	·156	
104	10 18·5	34 41	7·0, 7·5	286·3	206·72	·156	
105	,, 24·3	29 5	7·0, 8·0	225·4	131·48	·156	225°·3 : 131″·32 : 2 *n*.
				225·2	131·16	·175	
107	11 5·2	66 35	7·2, 7·5	43·6	70·04	·175	43°·75 : 69″·82 : 2 *n*. A and D <.
				43·9	69·60	·178	
108	,, 7·3	36 23	6·0, 7·0	70·2	137·56	·175	70°·1 : 137″·145 : 2 *n*. A > ; D <.
				70·0	136·73	·178	
109	,, 10·2	46 25	7·5, 8·0	257·2	81·35	·175	257°·2 : 81″·19 : 2 *n*. D <.
				257·2	81·03	·178	
111	,, 24·7	30 30	7·0, 9·0	33·6	67·32	·189	33°·3 : 67″·00 : 2 *n*.
				33·0	66·69	·192	
112	,, 49·5	19 59	8·0, 8·2	35·6	72·77	·189	
114	,, 55·1	37 17	7·5, 8·0	81·3	87·75	·189	81°·35 : 87″·435 : 2 *n*.
				81·4	87·12	·192	
116	,, 56·8	0 40	7·5, 8·0	180·5	74·23	·203	180°·35 : 74·55 : 2 *n*. A >.
				180·2	74·87	·208	

I. *Otto Struve's Catalogue of 256 Doubles between 32″ and 2′ Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
117	12 6·5	+82 16	6·0, 8·0	75·7	66·75	1914·203	75°·65 : 66·72 : 2 *n*. D <.
				75·6	66·69	·208	
119	,, 26·1	1 53	7·0, 8·0	289·4	49·80	·203	
121	13 6·0	62 46	6·5, 9·5	9·5	108·18	·211	P. XIII 12.
122	,, 9·4	57 13	7·0, 8·0	212·1	118·08	·211	211°·85 : 117°·86 : 2 *n*. A and D <.
				211·6	117·64	·214	
123	,, 23·8	65 15	6·4, 6·8	146·8	68·34	·216	
124	,, 26·7	24 44	7·0, 7·0	257·5	70·68	·216	257°·75 : 70″·52 : 2 *n*. A > ; D <.
				258·0	70·36	·233	
125	,, 42·7	39 2	5·2, 8·2	237·3	71·40	·214	
126	,, 45·7	21 47	6·3, 6·8	208·6	86·29	·216	P. XIII 219–20.
127	,, 48·7	68 49	6·5, 8·0	64·8	77·09	·216	64°·9 : 77″·125 : 2 *n*. A > ; D <.
				65·0	77·16	·233	
129	14 29·9	24 52	7·0, 7·5	67·8	78·49	·219	
130	,, 35·4	80 47	8·5, 9·0	299·8	48·58	·219	
131	,, 48·6	0 2	6·5, 7·3	212·7	88·83	·244	212°·65 : 88″·925 : 2 *n*. P. XIV 205·7. A <.
				212·4	89·02	·249	
132	,, 56·8	54 15	6·5, 7·0	342·4	40·55	·233	
133	,, 57·2	47 40	6·0, 8·5	156·2	35·46	·236	156°·3 : 35″·75 : 2 *n*. =OΣ 291 *rej*.
				156·4	36·04	·244	
135	15 8·6	28 19	8·0, 8·5	228·6	34·01	·244	228°·15 : 33″·85 : 2 *n*. D <.
				227·7	33·69	·249	
137	,, 12·7	51 19	6·5, 8·5	105·3	72·71	·246	105°·5 : 72″·925 : 2 *n*. P. XV 39. A and D >.
				105·7	73·14	·249	
138	,, 17·9	60 44	7·0, 7·2, 9·0	199·0	150·44	AB ·249	Triple.
,,	,, ,,	,,	,, ,,	165·2	82·04	AC ·249	
140	,, 27·6	8 57	7·7, 8·0	180·1	112·24	·249	
141	,, 36·5	57 47	7·0, 9·0	205·8	91·88	·252	
142	16 3·1	60 20	7·0, 9·0	264·6	104·95	·252	264°·9 : 104″·92 : 2 *n*.
				265·2	104·89	·255	
143	,, 5·3	70 32	6·5, 8·0	84·9	46·25	·252	85°·05 : 46″·44 : 2 *n*.
				85·2	46·63	·255	
144	,, 17·5	19 23	3·8, 8·2	236·5	41·06	·255	γ Herc.
145	,, 19·1	32 34	6·5, 9·0	18·7	34·74	·252	23 Herc. 18°·55 : 34″·72 : 2 *n*.
				18·4	34 70	·255	
146	,, 24·7	8 31	6·5, 7·0	71·9	58·12	·255	
149	,, 39·1	20 47	7·0, 7·5	135·3	98·35	·255	

I. *Otto Struve's Catalogue of* 256 *Doubles between* 32″ *and* 2′ *Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
150	16 54·8	+9 51	7·5, 8·0	164·0	77·92	1914·255	=Σ 2111 *rej.*
151	17 1·7	53 22	7·5, 8·5	171·8	80·59	1913·810	172°·3 : 80″·60 : 2 *n.* Dist. <.
				172·8	80·60	·813	
152	,, 14·4	21 53	7·0, 9·0	49·4	52·16	·810	
153	,, 21·4	39 20	6·5, 7·5	197·8	87·30	·808	197°·75 : 88″·61 : 2 *n.*
				197·7	89·92	·810	
157	,, 37·0	31 20	6·5, 7·5	109·2	114·21	·810	P. XVII 204. 109°·3 : 114″·08 : 2 *n.* Ang. > ; dist. <.
				109·4	113·95	·813	
159	,, 46·9	1 8	6·0, 7·0	237·4	81·97	·810	
161	,, 55·4	8 51	6·5, 8·0	77·3	64·34	·808	76°·6 : 63″·26 : 2 *n.*
				75·9	62·18	·810	
162	,, 55·7	2 56	5·6, 7·5	143·7	54·44	·805	67 Ophiuchi.
163	,, 55·2	62 38	7·0, 7·0	42·0	57·86	·808	41°·25 : 58″·24 : 2 *n.* Ang. < ; dist. >.
				40·5	58·62	·810	
164	,, 58·4	7 56	7·3, 8·2	2·9	50·12	·808	
165	18 1·1	4 33	7·4, 7·9	141·3	66·10	·813	
167	,, 13·3	4 32	7·5, 8·5	76·7	54·06	·805	77°·65 : 54″·25 : 2 *n.* Ang. > ; dist. < ?
				78·6	54·44	·808	
168	,, 23·6	4 46	7·5, 8·5	164·8	48·47	·805	Decl. 4° 36′ ?
170	,, 25·8	4 26	6·5, 8·0	5·6	100·95	·717	5°·3 : 101″·39 : 2 *n.*
				5·1	101·83	·728	
171	,, 29·5	38 45	6·5, 7·5	321·3	143·14	·717	P. XVIII 126–7. 321°·35 : 142″·90 : 2 *n.* Ang. and dist. <.
				321·4	142·67	·728	
172	,, 40·9	33 54	7·5, 8·0	5·7	64·64	·772	
174	,, 43·5	11 2	7·0, 8·0	159·2	107·16	·731	
176	,, 49·5	1 46	7·0, 7·0	114·6	97·70	·731	
177	19 8·0	16 41	6·7, 7·5	285·1	115·38	·712	285°·6 : 114″·46 : 2 *n.* Ang. and dist. >.
				286·1	113·54	·715	
178	,, 10·8	14 54	5·5, 7·0	* ...	90·38	·682	268°·0 : 90″·63 : 2 *n.*
				268·0	90·89	·712	
179	,, 15·0	12 12	5·8, 8·2	177·0	60·04	·712	28 Aquilæ. 175°·75 : 60″·24 : 2 *n.*
				174·5	60·44	·717	
180	,, 16·0	14 13	7·0, 8·0	* ...	80·41	·682	266°·5 : 80″·30 : 2 *n.*
				266·5	80·19	·712	
181	,, 16·0	26 28	6·2, 6·3	3·1	56·39	·717	3°·3 : 56″·40 : 2 *n.* Ang. > ; dist. <.
				3·5	56·40	·728	

* Ang. rejected—error in reading vernier.

I. *Otto Struve's Catalogue of 256 Doubles between 32″ and 2′ Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
182	19 24·0	+50 2	7·0, 7·5	304·5	70·64	1913·682	P. XIX 154. 304°·35 : 71″·38 : 2 *n.* Ang. >.
				304·2	72·12	·715	
185	,, 32·8	16 14	5·6, 7·8	81·1	90·38	·712	ϵ Sagittæ.
187	,, 32·5	46 12	7·0, 7·5, 7·2	286·1	64·48	AB ·731	Triple.
,,	,, ,,	,,	,,	253·9	130·02	AC ·731	
188	,, 39·2	37 25	7·0, 7·5	121·0	58·92	·731	
190	,, 40·4	47 0	7·5, 9·0	316·3	66·95	·772	
191	,, 42·1	34 46	6·0, 8·0	26·8	39·16	·717	P. XIX 278. 27°·2 : 38″·53 : 2 *n.*
				27·6	37·90	·728	
192	,, 42·6	32 50	6·0, 8·0	200·6	31·85	·717	200°·7 : 31″·47 : 2 *n.* Ang. >.
				200·8	31·09	·728	
194	,, 51·5	59 25	5·5, 8·5	359·1	75·28	·715	
195	,, 52·8	26 57	6·5, 7·0	136·1	92·90	·717	136°·2 : 92″·65 : 2 *n.*
				136·3	92·40	·728	
196	,, 58·2	40 33	7·0, 8·0	168·4	55·50	·731	
197	,, 58·5	49 49	5·3, 8·5	148·6	43·93	·717	26 Cygni. 148°·05 : 41″·90 : 2 *n.*
				147·5	39·87	·728	
198	20 1·5	7 15	7·0, 7·5	186·3	64·84	·731	
199	,, 1·8	35 20	7·0, 8·0	323·9	69·73	·731	
200	,, 1·2	64 21	6·5, 8·0	335·8	97·55	·728	65 Drac. 336°·55 : 98″·10 : 2 *n.*
				337·3	98·65	·846	
202	,, 9·3	6 17	6·9, 7·3	192·8	43·93	·717	P. XX 43–4.
203	,, 8·9	33 54	8·0, 9·0	37·7	90·45	·772	
205	,, 16·2	40 51	7·0, 8·5	319·1	45·57	·734	
206	,, 19·1	38 53	7·0, 8·2	256·4	43·17	·734	
207	,, 19·6	42 40	6·5, 8·0	63·4	94·17	·731	63°·05 : 93″·53 : 2 *n.* Dist. >.
				62·7	92·90	·734	
208	,, 31·4	46 31	7·5, 8·0	240·8	77·20	·772	
209	,, 40·3	12 20	7·1, 7·3	157·3	96·39	·778	
210	,, 45·0	5 10	6·0, 8·5	123·0	80·95	·734	
211	,, 46 8	58 22	6·5, 7·0	262·3	110·06	·734	262°·2 : 110″·48 : 2 *n.* Dist. >.
				262·1	110·91	·737	
212	,, 49·4	30 33	7·5, 9·0	153·5	65·89	·778	
213	,, 55·2	16 26	6·8, 9·0	36·5	70·76	·734	
214	21 0·1	41 14	6·0, 8·0	184·9	57·05	·742	P. XX 465.
215	,, 6·9	47 17	6·1, 6·9	189·5	134·49	·742	

I. *Otto Struve's Catalogue of* 256 *Doubles between* 32″ *and* 2′ *Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
216	21 10·2	+33 55	6·5, 7·0	46·5	102·99	1913·742	46°·8 : 102″·15 : 2 *n*.
				47·1	101·31	·764	
217	,, 16·3	52 38	6·5, 8·5	300·3	48·97	·742	300°·95 : 49″·06 : 2 *n*.
				301·6	49·15	·778	
220	,, 34·2	55 20	8·3, 8·3	10·8	41·55	·778	
221	,, 36·3	42 49	5·0, 9·5	253·4	56·39	·742	75 Cygni. 253°·85 : 56″·93 : 2 *n*. Dist. <.
				254·3	57·48	·745	
222	,, 39·2	6 39	7·0, 7·5	258·0	89·91	·753	258°·0 : 88″·75 : 2 *n*.
				258·0	87·60	·756	
223	,, 39·2	9 25	2·5, 8·8	320·5	143·11	·753	ε Pegasi. 320°·5 : 142″·56 : 2 *n*. Dist. <.
				320·5	142·01	·756	
224	,, 41·0	15 17	7·5, 8·5	4·6	58·64	·778	Ang. > ?
225	,, 52·4	3 40	7·0, 8·0	286·2	76·45	·753	286°·4 : 76″·83 : 2 *n*. Dist. <.
				286·6	77·29	·780	
226	,, 50·9	67 37	7·0, 8·0	245·8	77·04	·756	246°·05 : 76″·24 : 2 *n*.
				246·3	75·45	·778	
227	,, 52·7	11 27	7·5, 8·0	32·1	81·74	·780	32°·45 : 79″·83 : 2 *n*.
				32·8	77·92	·788	
228	,, 56·8	4 18	7·5, 9·0	26·8	76·49	·780	26°·45 : 76″·95 : 2 *n*. Ang. > ; Dist. <.
				26·1	77·41	·788	
229	22 0·9	59 23	6·5, 9·5	71·7	184·60	AC ·742	P. XXI 401. Quad.
			7·5, 6·7				
,,	,, ,,	,,	,, ,,	36·8	237·60	AD ·742	B too faint for illum.
230	,, 9·8	39 59	7·5, 8·5	160·5	46·09	·780	160°·25 : 45″·50 : 2 *n*.
				160·0	44·92	·788	
231	,, 17·7	9 26	7·0, 8·0	110·0	91·47	·761	
232	,, 18·6	3 18	8·7, 9·0	192·3	65·89	·780	
234	,, 22·8	49 12	7·3, 7·7	134·3	35·87	·780	
236	,, 33·5	72 21	7·0, 7·5	136·7	42·29	·756	
237	,, 47·2	40 48	6·5, 7·5	337·4	82·57	·756	
238	,, 49·1	67 27	6·5, 7·0	280·3	70·59	·756	280°·35 : 69″·70 : 2 *n*.
				280·4	68·81	·778	
239	,, 50·5	35 49	5·5, 8·0	244·1	51·17	·761	243°·7 : 50″·91 : 2 *n*. Dist. <.
				243·3	50·65	·780	
241	,, 53·6	11 30	7·2, 7·5	161·3	84·29	·780	
242	23 0·5	46 23	7·0, 8·0	30·8	80·00	·761	
243	,, 5·4	36 18	6·5, 7·0	318·7	67·60	·761	318°·8 : 67″·70 : 2 *n*. Dist. <.
				318·9	67·79	·780	

I. *Otto Struve's Catalogue of* 256 *Doubles between* 32″ *and* 2′ *Distance*—continued.

OΣΣ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
244	23 15·0	+47 49	6·0, 9·0	300·3	86·51	1913·761	P. XXIII 51. 300°·45 : 85″·90 : 2 *n*.
				300·6	85·27	·788	Ang. > ; dist. <.
245	,, 21·0	27 2	7·0, 7·5	194·1	63·12	·761	
246	,, 23·0	23 2	7·5, 8·0	114·1	86·83	·780	114°·45 : 86″·87 : 2 *n*. Ang. < ;
				114·8	86·91	·788	dist. >.
248	,, 41·1	50 7	7·5, 9·5	140·3	52·92	·788	
251	,, 48·5	50 58	6·5, 9·0	200·8	43·44	·761	P. XXIII 223. 201°·1 : 43″·92 : 2 *n*.
				201·4	44·41	·788	Ang. and dist. <.
252	,, 49·8	28 54	6·5, 7·5	143·3	111·36	·761	
253	,, 55·8	69 0	8·0, 8·2	353·6	100·12	·788	
254	,, 56·2	59 48	6·3, 7·7	268·9	58·54	·761	
255	0 0·2	15 47	8·0, 8·2	336·6	90·86	·788	336°·9 : 90″·67 : 2 *n*. Dist. <.
				337·2	90·48	·791	
256	,, 2·8	30 49	7·0, 7·0	115·6	104·60	·761	115°·55 : 105″·22 : 2 *n*. Dist. <.
				115·5	105·84	·788	

II. *W. Struve's Appendix I.*

Σ I.	R.A. 1900.	Decl 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
1	0 41·0	+30 24	6·7, 6·7	52·8	46″25	1913·824	P.O 175–6. 52°·45 : 46″·18 : 2 *n*.
				52·1	46·12	·832	
2	1 0·7	4 22	5·9, 6·8	82·7	34·14	·824	= Σ 90.
				83·8	33·37	·832	77 Piscium. 83°·25 : 33″·75 : 2 *n*.
3	,, 9·3	− 8 28	5·1, 7·0	331·4	50·12	·832	37 Ceti.
4	,, 50·2	+36 46	6·0, 6·0	299·1	188·95	·890	299·65 : 189·02 : 2 *n*. 56 And. and
				300·2	189·08	·895	P. I 203. Ang. > ; dist. <.
5	2 31·2	24 13	6·1, 7·1	273·6	38·63	·890	30 Ari. and P. II 128.
6	3 3·8	7 5	7·0, 7·0	163·1	80·78	·906	
7	,, 25·0	27 24	6·9, 7·4	232·5	44·35	·906	
8	,, 41·5	23 48	3·8, 7·0	289·3	117·19	·906	η Tauri.
9	4 19·4	22 4	5·0, 6·0	173·0	340·59	·923	κ^1, κ^2 Tauri.
10	,, 22·8	15 44	4·7, 5·0	346·5	337·42	·923	θ^1, θ^2 ,,
11	,, 33·5	15 43	5·2, 5·7	193·0	432·60	·923	σ^1, σ^2 ,, Dist. <.
12	,, 55·5	26 32	6·0, 7·0	158·6	78·42	·961	158°·45 : 78″·675 : 2 *n*.
				158·3	78·93	·966	
13	,, 57·4	58 50	5·0, 6·0	7·5	180·06	·961	11, 12 Cam. 7°·5 : 179″·875 : 2 *n*.
				7·5	179·69	·966	
14	5 26·9	− 0 23	2·0, 6·8	359·4	52·47	·977	δ Orionis.

II. *W. Struve's Appendix I.*—continued.

Σ I.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
15	5 30·3	−6 4	5·6, 6·5	223·0	36·17	1913·977	=Σ 747.
16	,, 30·4	5 27	4·8, 6·1	91·9	52·60	·977	θ^2 Orionis.
17	,, 30·4	5 27	4·8, 5·2	314·3	134·83	·977	θ^2, θ^1 Orionis.
18	10 11·0	24 0	3·8, 6·0	340·7	323·40	1914·156	ζ and 35 Leonis. A >; D <.
19	11 22·8	3 25	5·0, 7·0	175·1	90·67	·178	174°·7 : 90″·765 : 2 *n*. τ Leonis. A <; D >.
				174·3	90·86	·189	
20	,, 49·9	47 2	6·1, 6·5 AC	113·8	63·13	·178	65 Urs. Maj. AB=Σ 1579.
21	12 23·8	26 29	4·8, 6·0	250·7	145·00	·203	17 Com. Ber. and P. XII 96.
22	,, 40·4	14 55	6·5, 7·0	191·1	32·23	·208	=Σ 1678. A >.
23	,, 47·2	17 37	5·3, 6·1	49·2	195·17	·208	32, 33 Comæ Ber.
24	13 5·5	39 2	5·5, 5·9	297·4	283·88	·211	15, 17 Can. Ven. D >.
				297·0	284·93	·214	297°·2 : 284″·405 : 2 *n*.
25	,, 9·6	67 48	5·9, 6·3	296·4	178·40	·211	
26	14 12·7	51 50	4·9, 7·5	33·2	38·52	·219	ι Boötis.
27	15 11·5	33 42	3·2, 7·4	78·4	104·43	·244	δ Boötis. 78°·3 : 104″·655 : 2 *n*.
				78·2	104·88	·249	
28	,, 20·7	37 43	4·0, 6·5	171·8	108·24	·249	μ^1, μ^2 Boötis.
29	16 18·7	33 56	4·8, 5·1	165·2	365·97	1913·695	ν^1, ν^2 Coronæ. Dist. >.
30	,, 33·7	53 10	5·0, 5·0	15·0	89·59	·638	14°·45 : 90″·00 : 2 *n*. 16, 17 Drac.
				13·9	90·41	·690	
31	,, 35·6	4 24	6·0, 7·0	229·9	69·53	·641	36, 37 Herculis.
32	,, 55·0	47 31	7·0, 7·1	262·8	114·67	·638	
33	,, 59·2	13 44	5·8, 6·3	116·2	297·25	·635	116°·0 : 299″·64 : 3 *n*. P. XVI 283–5. Dist. <.
				115·8	299·67	·690	
				116·0	301·99	·693	
34	17 29·9	9 39	5·6, 7·3	191·2	40·78	·635	53 Ophiuchi.
35	17 30·2	55 15	4·6, 4·6	312·7	61·68	·635	ν^1, ν^2 Drac. Ang. reversed.
36	18 22·5	58 46	4·7, 7·1	22·0	90·16	·638	39 Drac. 21°·45 : 89″·98 : 2 *n*.
				20·9	89·80	·747	
37	,, 41·1	39 30	4·6, 4·9	172·3	208·93	·624	ϵ^1, ϵ^2 Lyræ. 172°·5 : 208″·37 : 3 *n*.
				172·4	208·48	·747	
				172·7	207·71	·778	
38	,, 41·3	37 30	4·2, 5·5	147·7	43·92	·633	ζ Lyræ. 148°·4 : 44″·03 : 2 *n*.
				149·1	44·15	·690	
39	,, 46·3	33 15	3·0, 6·7	148·9	46·20	·633	β Lyræ. 148°·45 : 45″·76 : 2 *n*.
				148·0	45·32	·690	
40	19 13·7	0 15	6·0, 6·2	136·2	425·64	·633	24 Aquilæ and P. XIX 55. 136°·2 : 424″·61 : 2 *n*.
				136·2	423·58	·753	

II. *W. Struve's Appendix I.*—continued.

Σ I.	R.A. 1900. h m	Decl. 1900. ° ′	Mags.	Pos. Angle. °	Distance. ″	Epoch.	Remarks.
41	19 19·8	+16 45	5·9, 6·7	78·9	339·03	1913·641	23 Sagittæ. 78°·7 : 338″·18 : 2 *n*. Dist. <.
				78·5	337·34	·753	
42	,, 24·6	24 30	4·4, 5·7	28·2	410·22	·641	6, 8 Vulp. 28°·0 : 410″·63 : 3 *n*. Dist. <.
				27·7	409·24	·682	
				28·0	412·43	·693	
43	,, 26·7	27 45	3·0, 5·3	55·7	34·50	·633	β Cygni.
44	,, 32·0	59 54	5·2, 7·2	287·7	76·43	·638	
45	,, 37·5	− 8 33	6·5, 6·9	146·2	97·20	·682	A >; D <.
46	,, 39·2	+50 18	5·1, 5·3	133·7	39·42	·682	16 Cyg. 134°·4 : 38″·99 : 3 *n*. Ang. >; Dist. <.
				135·4	39·14	·690	
				134·0	38·40	·693	
47	,, 49·3	− 8 29	5·2, 6·2	168·9	36·89	·682	169°·8 : 35″·92 : 3 *n*. 57 Aquilæ. Ang. >.
				170·2	35·00	·693	
				170·4	35·88	·695	
48	,, 49·0	+20 5	6·7, 6·8	148·2	42·79	·695	P. XIX 320–1.
49	20 5·5	20 37	6·0, 8·3, 7·1	325·8	11·82	AB ·709	θ Sagittæ. Triple. (AB = Σ 2637).
,,	,, ,,	,,	,, ,,	223·8	79·91	AC ·709	Ang. >; dist. <.
50	,, 10·5	46 26	3·7, 5·0	323·0	340·04	·704	o^1, o^2 Cygni. 323°·05 : 338″·50 : 2 *n*. Ang. >.
				323·1	336·97	·747	
51	,, 12·1	−12 50	3·2, 4·2	291·2	378·23	·695	α^1, α^2 Capr. 291°·15 : 377″·98 : 2 *n*. Dist. <.
				291·1	377·73	·747	
52	,, 15·4	15 5	2·5, 6·0	267·5	206·08	·701	β Capricorni.
53	,, 33·4	+31 13	6·0, 6·1	175·3	180·00	·704	48 Cyg. and P. XX 243. Ang. and Dist. <.
54	21 5·5	9 44	4·2, 5·7	152·2	354·80	·704	γ, 6 Equulei. Dist. >.
55	,, 19·7	36 56	6·0, 6·6	302·7	365·99	·704	
56	,, 32·7	6 10	6·0, 7·4	349·1	39·92	·704	3 Pegasi. 349°·3 : 39″·98 : 2 *n*.
				349·5	40·05	·753	
57	,, 32·5	66 16	6·5, 6·5	25·5	180·89	·712	Dist. <; Ang. >.
58	22 25·4	57 54	3·0, 5·3	192·0	42·06	·712	δ Cephei. 191°·9 : 41″·66 : 3 *n*.
				192 2	41·91	·715	
				191·5	41·02	·756	
59	,, 59·9	− 8 5	5·6, 7·0	146·9	258·20	·704	83, 84 Aquarii. Dist. >.
60	23 30·7	+59 54	6·4, 6·5	29·7	241·60	·712	Dist. >.

III. *Wide Struvian Pairs affected with Proper Motion.*

Σ.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
30	0 21·8	+49 26	6·8, 8·7	303·8	17·51	1913·824	302°·7 : 17″·98 : 2 *n.*
				301·6	18·46	·832	
80	,, 54·3	0 15	7·0, 8·2	321·2	22·90	·824	321°·5 : 22″·84 : 2 *n.*
				321·8	22·78	·832	
117	1 18·8	67 36	4·4, 8·4	109·3	27·35	1914·096	ψ Cass. AC.
				110·6	26·21	·110	109°·95 : 26″·78 : 2 *n.*
142	,, 34·5	14 45	8·2, 8·4	350·5	14·28	1913·868	351°·9 : 13″·96 : 2 *n.*
				353·3	13·64	·879	
143	,, 34·7	33 51	7·7, 9·0	319·5	37·88	·868	318°·95 : 37″·85 : 2 *n.*
				318·4	37·82	·879	
175	,, 45·4	20 37	7·9, 8·4	349·2	17·26	·868	348°·8 : 17″·38 : 2 *n.*
				348·4	17·51	·879	
190 *rej.*	,, 52·0	40 54	7·5, 8·5	78·5	25·57	·879	77°·65 : 25″·48 : 2 *n.*
				76·8	25·38	·882	
197	,, 55·2	34 49	7·2, 8·2	232·4	27·16	·879	232°·15 : 26″·74 : 2 *n.*
				231·9	26·33	·882	
343	3 8·8	83 41	8·0, 8·8	324·6	29·19	1914·096	324°·55 : 28″·74 : 2 *n.*
				324·5	28·30	·110	
434	,, 37·4	38 4	7·0, 7·8	83·8	30·83	1913·890	84°·55 : 30″·65 : 2 *n.*
				85·3	30·46	·895	
=II 1 518	4 10·7	− 7 47	4·0, 9·2	105·2	83·31	·923	40 Erid.
				104·9	81·91	·950	105°·05 : 82″·61 : 2 *n.*
613	,, 51·6	+43 59	7·7, 8·7	103·9	16·62	·950	103°·75 : 16″·31 : 2 *n.*
				103·6	15·99	·955	
634	5 6·1	79 8	4·5, 7·9	34·6	11·55	·977	34°·0 : 11″·515 : 2 *n.*
				33·4	11·48	·986	
II 3	,, 12·1	40 1	5·2, 8·7	5·0	141·62	·966	λ Aur. 4°·65: 141″·945: 2*n.*
				4·3	142·27	·977	
735	,, 28·0	− 6 34	8·2, 9·0	350·6	45·36	·988	30°·50 : 45″·645 : 2 *n.*
				350·4	45·93	·994	
782	,, 37·8	0 2	7·8, 8·3	307·2	42·64	·988	307°·15 : 42″·45 : 2 *n.*
				307·1	42·26	·994	
853	6 3·5	+11 41	7·9, 8·2	356·6	29·95	1914·011	356°·5 : 30″·365 : 2 *n.*
				356·4	30·78	·014	
859	,, 4·3	5 40	8·2, 8·5	245·7	37·31	·011	245°·5 : 38″·10 : 2 *n.*
				245·3	38·89	·014	
861	,, 4·9	30 42	7·8, 8·2	16·3	65·23	·096	17°·2 : 65″·20 : 2 *n.*
				18·1	65·17	·115	

III. *Wide Struvian Pairs affected with Proper Motion*—continued.

Σ	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
1025	7 4·6	+55 58	7·5, 7·8	134·5	25·06	1914·090	134°·75 : 25″·23 : 2 *n.*
				135·0	25·40	·093	
1091	,, 22·3	50 10	8·2, 8·7	329·5	28·17	·096	329°·3 : 28″·71 : 2 *n.*
				329·1	29·25	·123	
II 4	,, 28·2	32 7	2·7, 9·5	163·8	71·57	·090	Castor AC.
				163·8	73·09	·093	163°·8 : 72″·33 : 2 *n.*
1234	8 25·4	55 41	7·0, 8·3	67·0	22·78	·131	67°·9 : 22″·46 : 2 *n.*
				68·8	22·14	·140	
1236	,, 25·2	32 16	8·0, 8·5	112·6	37·18	·131	112°·05 : 37″·72 : 2 *n.*
				111·5	38·26	·140	
1263	,, 38·6	42 3	7·6, 8·2	22·1	64·02	·140	21°·8 : 64″·02 : 2 *n.*
				21·5	64·02	·142	
1281	,, 42·4	0 24	7·3, 8·3	318·1	37·62	·140	318°·75 : 37″·78 : 2 *n.*
				319·4	37·94	·142	
1321	9 7·7	53 7	7·4, 7·4	71·6	19·48	·148	70°·45 : 19″·445 : 2 *n.*
				69·3	19·41	·151	
1329	,, 10·6	−0 49	8·3, 8·5	251·2	20·17	·142	251°·95 : 20″·295 : 2 *n.*
				252·7	20·42	·145	
1402	,, 58·2	+55 59	6·8, 8·0	101·5	26·33	·145	101°·35 : 26″·27 : 2 *n.*
				101·2	26·21	·151	
II 6	10 3·1	12 28	1·5, 8·4	307·0	176·70	·153	Regulus.
1472	,, 41·7	13 34	7·8, 8·5	38·1	37·35	·156	38°·0 : 37″·485 : 2 *n.*
				37·9	37·62	·175	
1516	11 8·7	74 0	7·0, 7·5	99·5	24·40	·156	99°·35 : 24″·675 : 2 *n.*
				99·2	24·93	·175	
II 7	,, 42·8	20 47	4·7, 8·4	355·5	74·24	·175	93 Leonis.
1602	12 2·3	69 38	7·5, 9·0,	178·9	16·12	·192	179°·0 : 16″·65 : 2 *n.*
				179·1	17·19	·203	
1604	,, 4·3	−11 18	6·5, 9·0	AB 90·6	11·10	·203	Triple.
			8·0				
,,	,, ,,	,,	,, ,,	AC 89·5	31·53	·203	
1607	,, 6·5	+36 39	7·8, 8·3	4·6	29·77	·192	4°·65 : 29″·32 : 2 *n.*
				4·7	28·87	·203	
1664	,, 33·2	−10 58	7·7, 8·8	238·4	23·60	·211	
σ434	13 9·7	10 50	7·5, 8·5	48·3	75·89	·211	P. XIII 25.
2241	17 43·7	+72 12	4·0, 5·2	14·4	30·71	1913·690	ψ^1 Drac.
2398	18 41·8	59 28	8·2, 8·7	151·7	17·20	·849	152°·0 : 16″·94 : 2 *n.*
				152·3	16·68	·851	

III. *Wide Struvian Pairs affected with Proper Motion*—continued.

	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Dis. tance.	Epoch.	Remarks.
	h m	° ′		°	″		
2420	18 49·7	+59 17	4·6, 8·1	332·9	32·10	1913·843	*o* Drac. 332°·7 : 32″·07 : 2 *n.*
				332·5	32·04	·846	
2456	19 2·3	38 22	8·2, 8·2	181·1	23·03	·849	181°·45 : 22″·84 : 2 *n.*
				181·8	22·65	·851	
2486	,, 9·5	49 40	5·8, 6·1	216·0	9·58	·849	216°·15 : 9″·32 : 2 *n.*
				216·3	9·07	·851	
2515	,, 20·3	21 19	8·0, 9·0	32·3	9·99	·849	31°·8 : 9″·78 : 2 *n.*
				31·3	9·58	·851	
2549	,, 30·0	63 6	7·7, 8·9	287·1	24·23	·851	AB.
Triple	,, ,,	,,	,, 7·7	274·7	51·97	·851	AC.
2612	,, 56·5	6 38	7·8, 8·8	53·8	39·34	·849	
2646	20 9·1	− 6 21	7·0, 8·8	46·2	22·14	·849	46°·55 : 21″·70 : 2 *n.*
				46·9	21·26	·851	
2708	,, 34·9	+38 17	7·0, 8·7	329·5	31·15	·849	329°·05 : 31″·18 : 2 *n.*
				328·6	31·21	·851	
2734	,, 49·3	12 42	8·2, 8·7	201·7	24·24	·849	
2754	21 1·4	12 46	8·0, 8·7	297·2	32·93	·849	298°·7 : 32″·69 : 2 *n.*
				300·2	32·36	·860	
2758	,, 2·3	38 14	5·3, 5·9	128·9	23·60	·849	61 Cygni.
				129·5	23·47	·860	129°·2 : 23″·53 : 2 *n.*
II 11	,, 17·5	19 23	4·5, 8·6	312·8	35·64	·704	1 Pegasi.
				311·9	36·42	·860	312°·35 : 36″·03 : 2 *n.*
2803	,, 26·6	52 30	7·4, 9·0	287·1	25·12	·849	
2822	,, 39·6	28 18	4·0, 7·0	53·3	205·58	·742	μ Cygni AC.
				52·9	205·34	·745	53°·1 : 205″·46 : 2 *n.*
II 12	23 10·6	− 9 38	4·5, 8·5	311·6	49·99	·712	ψ^1 Aquarii.
				312·5	49·49	·849	312°·05 : 49″·74 : 2 *n.*
3041	,, 42·8	+16 31	7·3, 8·1	351·9	65·54	·868	A–BC.
Triple				352·4	65·23	·879	352°·15 : 65″·38 : 2 *n.*

IV. *Miscellaneous Wide Doubles* (*Herschel, etc.*).

Syn.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Dis- tance.	Epoch.	Remarks.
	h m	° ′		°	″		
Hh 11	0 34·8	+56 0	2·5, 9·0	280·3	62·05	1913·813	α Cassiop.
				280·8	63·19	·819	280°·55 : 62″·62 : 2 *n.*
h 323	,, 35·6	− 4 54	6·2, 8·7	289·5	65·48	·819	P. O 146.
...	3 14·7	+48 43	6·0, 8·5	327·6	205·84	·890	P. III 28 ("Cel. Obj.")
...	,, 34·5	+59 39	6·0, 9·0	34·4	55·39	·890	P. III 97 ,,

IV. *Miscellaneous Wide Doubles (Herschel, etc.)*—continued.

Syn.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Remarks.
	h m	° ′		°	″		
Hh 138	4 51·4	− 5 20	6·0, 8·0	74·5	67·83	1913·923	62 Eridani.
				74·7	66·75	·950	74°·6 : 67″·29 : 2 *n*.
,, 166	5 18·6	+17 17	5·7, 8·2	271·2	83·75	·977	111 Tauri.
				270·6	82·74	·986	270°·9: 83″·245: 2 *n*. D<.
h 702	,, 24·7	− 2 2	8·0, 9·0	152·0	24·11	1914·085	Error in H's quadrant? Fr. est. 150° in 1885.
Hh 199	,, 40·4	−22 28	4·0, 6·5	350·8	94·92	·085	γ Leporis.
,, 209	,, 52·5	+44 27	7·1, 9·6	327·2	35·21	1913·988	327°·4 : 34″·925 : 2 *n*.
				327·6	34·64	·994	
,, 223	6 21·8	+20 51	6·0, 8·0	205·5	29·51	1914·011	15 Gemin.
				205·5	30·02	·014	205°·5 : 29″·765 : 2 *n*.
,, 239	,, 32·0	−18 35	5·7, 8·0	262·4	17·76	·088	ν^1 Can. Maj.
,, 246	,, 37·8	+25 13	3·0, 9·5	94·0	108·05	·011	ϵ Gemin.
				93·4	109·45	·014	93°·7 : 108″·75 : 2 *n*.
h 3945	7 12·4	−23 8	6·5, 8·0	61·7	27·54	·088	
Hh 264	,, 21·7	+22 21	7·6, 9·3	275·2	37·18	·090	275°·6 : 37″·085 : 2 *n*.
				276·0	36·99	·093	
,, 269	,, 30·1	−23 15	6·0, 6·0	109·4	11·29	·088	P. VII 149–7.
...	,, 43·1	−15 44	6·4, 6·4	311·2	130·52	·088	Ll. 15219 ("Cel. Obj.").
Hh 284	,, 53·2	+ 2 30	6·0, 8·5, 9·0	AB 74·9	88·13	·090	75°·3 : 88″·355 : 2 *n*.
				75·7	88·58	·093	14 Can. Min. Trip.
,,	,, ,,	,,	,,	AC 152·4	115·92	·090	152°·35 : 116″·335 : 2 *n*.
				152·3	116·75	·093	
,, 303	8 38·7	− 6 52	5·5, 8·1	308·9	79·63	·140	31 Monoc.
				309·5	78·42	·145	309°·2 : 79″·025 : 2 *n*.
,, 313	,, 51·5	+32 50	5·6, 8·9	295·8	91·37	·142	64 Cancri.
				296·4	90·10	·145	296°·1 : 90″·735 : 2 *n*.
,, 314	,, 55·9	+28 18	6·0, 8·0	326·7	102·60	·142	67 Cancri.
				325·2	102·66	·145	325°·95 : 102″·63 : 2 *n*.
,, 333	9 24·1	− 2 19	5·5, 8·5	3·9	65·61	·148	τ^1 Hydræ.
				3·6	66·37	·153	3°·75 : 65″·99 : 2 *n*.
5 605	,, 48·9	+ 5 25	6·7, 9·0	292·3	52·60	·148	9 Sextantis.
				289·8	53·19	·153	291°·05 : 52″·89 : 2 *n*.
Hh 368	11 12·0	− 6 36	7·0, 9·0	97·5	59·58	·175	
...	,, 44·0	+15 8	2·5, 8·0	202·6	264·07	·178	β Leonis ("Cel. Obj.").
				202·5	263·88	·189	202°·55 : 263″·957 : 2 *n*. A and D >.
Hh 395	12 17·5	+26 24	4·5, 8·5	167·3	64·34	·203	12 Comæ.
				166·8	63·77	·208	167°·05: 64″·055: 2 *n*. D>.

IV. *Miscellaneous Wide Doubles (Herschel, etc.)*—continued.

Syn.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Dis. tance.	Epoch.	Remarks.
	h m	° ′		°	″		
Hh 396	12 24·7	−15 57	3·0, 8·5	213·5	24·36	1914·211	δ Corvi.
,, 432	13 56·6	+ 2 2	4·0, 9·0	288·1	80·85	·216	τ Virginis.
				288·6	80·78	·233	288°·35 : 80″·815 : 2 *n*.
,, 598	18 59·7	− 4 11	6·0, 7·5	207·3	37·90	1913 ·734	15 Aquilæ.
...	19 35·0	−16 31	5·5, 8·0	47·4	46·82	·805	54 Sagittarii ("Cel. Obj.").
h 1470	20 0·0	+38 2	7·0, 8·0	338·0	29·68	·794	A and D <.
Hh 671	,, 6·2	− 0 25	7·5, 8·0	206·0	54·44	·794	P. XX 12.
,, 682	,, 13·6	−19 26	5·0, 8·3	180·9	55·24	·753	σ Capri.
S 749	,, 22·3	− 2 26	7·2, 8·2	189·6	60·28	·794	P. XX 139, 140.
Hh 689	,, 24·1	−18 55	6·3, 6·8	239·4	22·46	·753	o^2 Capri.
,, 694	,, 28·2	+48 53	6·5, 8·5	277·5	60·48	·753	277°·8 : 60″·50 : 2 *n*.
				278·1	60·53	·791	P. XX 199.
...	,, 56·0	+18 56	6·2, 8·7	332·0	47·71	·742	332°·2 : 47″·38 : 2 *n*.
				332·4	47·05	·745	Ll. 40682. D <.
S 781	21 8·5	+ 6 48	6·5, 7·0	352·4	184·71	·761	352°·4 : 184″·92 : 2 *n*.
				352·4	185·14	·791	
h 5524	,, 56·1	−16 6	6·0, 9·0	313·3	179·18	·791	
Hh 775	22 38·3	+29 42	3·0, 10·0	339·5	91·12	1914·088	η Pegasi.
,, 781	,, 44·3	−14 7	5·0, 9·0	294·6	132·74	1913·753	τ^2 Aquarii.
				295·3	131·98	·788	

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Observations of the Region of Nova No. 2 Persei.

By C. R. D'Esterre.

Recently in the course of tabulating the results of photographic observations of the variable star 11, 1913 Persei (*A.N.*, 4648), a column was added for notes relating to Nova No. 2. As some further plates are required for the determination of the period of the variable, and an interesting degree of variability in the Nova is satisfactorily established by duplicate exposures with the portrait lenses and confirmed by the reflector, the observations of the Nova have been carefully checked and drawn up independently as follows:—

Plate.	Gr. Mean Time. 1911.	h	Limit.	Nova. m
D18	Sept. 22	11·2	*f*	12·4
D111	Dec. 11	8·2	*d*	11·8
	1912.			
D172	Apr. 19	9·2	*c*	12·4
V281	Aug. 13	14·8	*f*	12·3
Da26	Nov. 1	9·2	*f*	12·6
R138	3	11·8	$<g$	12·2
	1913.			
Da62	Jan. 12	12·7	*f*	11·8
Da67	31	13·5	*d*	12·6
Da73	Feb. 11	11·8	*d*	11·8
Da80	23	8·6	*d*	12·0
V390	Mar. 8	10·1	$<g$	11·8
Da90	8	10·1	*g*	12·0
R169	11	8·6	$<g$	11·7
R170	19	9·2	$<g$	11·9
R172	25	8·4	$<g$	11·7
Da185	Aug. 2	12·7	*d*	invis.
V490	3	14·2	$<g$	12·65
Da190	3	14·2	$<g$	12·7
D299	3	14·2	*f*	12·55
V512	29	12·4	$<g$	12·8
Da214	29	12·4	*g*	12·65
R255	Sept. 9	12·8	$<g$	12·35
Da223	9	12·8	$<g$	12·4
D310	9	12·8	*g*	12·45

Plate.	Gr. Mean Time. 1913.	h	Limit.	Nova. m
Da235	Sept. 24	15·2	*f*	12·35
V539	27	11·6	$<g$	12·65
Da244	27	11·6	$<g$	12·45
R325	Nov. 3	11·6	$<g$	12·55
Da267	8	12·7	*d*	12·6
V568	22	9·5	$<g$	12·0
Da273	22	9·5	$<g$	12·0
Da285	Dec. 28	8·2	*f*	12·0
	1914.			
V588	Jan. 17	11·8	$<g$	12·0
Da293	17	11·8	$<g$	12·0
Da298	22	12·0	$<g$	12·4
Da302	23	11·2	*g*	12·35
R376	Feb. 4	11·8	$<g$	12·5
V606	18	10·5	$<g$	13·2
Da312	18	10·5	$<g$	13·25
R387	22	9·2	$<g$	12·25
Da317	22	9·2	*f*	12·3
Da323	24	10·3	*f*	12·4
R395	26	8·2	$<g$	12·35
V624	27	9·9	$<g$	12·75
Da335	27	9·9	$<g$	12·6
R406	Mar. 16	10·9	$<g$	12·8
V637	16	10·9	$>g$	12·9
Da350	16	10·9	*g*	12·8

The mean magnitude of the Nova from these observations is $12^m{\cdot}3$, and the range of variation $11^m{\cdot}7$ to $13^m{\cdot}2$; the fluctuations appear to be irregular and rapid, and the general condition particularly disturbed about the present time. All exposures since 1912 November 1 have been made on Griffin "Professional" plates.

The magnitudes of the comparison stars for the Nova have been measured from plates

R387, 1914 February 22, $9^h{\cdot}2$, Exp 20 min., Region of the Nova,
R388, ,, ,, $9^h{\cdot}8$, ,, 20 ,, Region of the North Pole,

as follows:—

$a = 10{\cdot}5$, $b = 12{\cdot}2$, $c = 12{\cdot}4$, $d = 12{\cdot}6$, $e = 12{\cdot}9$, $f = 13{\cdot}1$, $g = 13{\cdot}4$.

The method employed is as follows. The pole plate is placed on an examining frame face up and the variable plate laid face

down upon it, so that the particular star to be measured can be viewed with a power of 16 diameters as a close companion to a star near the pole whose magnitude has been determined at Harvard or Greenwich. If a polar star can be found similarly placed in relation to the centre of the field to the one being measured and equal to it in diameter and intensity, a magnitude is directly deducible; otherwise a polar pair is selected slightly brighter and fainter than the star to be measured: this latter is placed between them and its magnitude deduced from the known difference between the comparison pair. When all the variable star comparisons have been gone over, the plates are reversed, field plate below face up and pole above, and the magnitudes redetermined; the accepted magnitudes are the mean of the two sets of readings.

Independent sets of measures of the same pairs of plates made at considerable intervals of time show a mean error for any given star of $0^{m}\cdot1$, which is thought to be entirely satisfactory where a large number of plates are being passed in review as rapidly as is consistent with a due appreciation of the dangers which beset the formation of faint star images.

The magnitudes of the faint stars 1 to 5 in the immediate neighbourhood of the Nova have also been measured from the above pair of plates as follows:—

$$1 = 15\cdot^{m}7, \quad 2 = 16\cdot^{m}0, \quad 3 = 16\cdot^{m}3, \quad 4 = 16\cdot6, \quad 5 = 17\cdot^{m}2,$$

and the components of the double star immediately *np* the Nova from plate R325, 1913 November 3, $11^{h}\cdot6$, Exp. 2 hours, Region of the Nova, by means of a measuring scale standardised against the above five stars as follows:—component *sf* $18^{m}\cdot7$, *np* $19^{m}\cdot0$

The occurrence of this pair so near the Nova is interesting; in my experience such close pairs are extremely rare amongst very faint stars.

The following counts of stars have been made over plates R325, and V490, 1913 August 3, $14^{h}\cdot2$, Exp. 45 minutes:—

R325, an area of 4 square degrees centred on the Nova is covered by 1700 squares on a photographic measuring scale; counts of the stars in eight sets of ten squares evenly distributed over the whole area give a mean of nine stars per square = 15,300 stars down to a mean magnitude of 18·5 over the area covered. The number of B.D. stars included is 95; of these the number known to vary by more that one magnitude is one: ratio of peculiar stars, 1 in 95. Amongst the remaining 15,200 stars the superpositions of my own plates have only brought to light the variable 11, 1913, and the variability of Nova No. 2: ratio of peculiar stars 1 in 7600. Over an area of about $\frac{1}{4}$ square degree in the immediate neighbourhood of the Nova it has also been possible to check my plates against the reproductions of those obtained with the Crossley Reflector between 1901 November and 1902 February (*Astrophysical Journal*, vol. xvi., Plates XI. to XV.): the number of stars down to $19^{m}\cdot0$ is about 1500, but in no case has any marked change in brightness occurred.

V490 covers the region (1855) $2^h\ 56^m$ to $3^h\ 39^m$ and $+40^\circ\ 50'$ to $46^\circ\ 20'$. On the measuring scale 4400 squares cover an area of approximately 40 square degrees: the stars in eight sets of ten squares evenly distributed over the plate show a mean of 7·9 per square = 34,700 stars down to $15^m{\cdot}0$ over the whole area. The number of B.D. stars included is 833, and of these three are known to vary more than one magnitude: ratio of peculiar stars 1 in 277. My own superpositions have failed to detect any certain case of variation amongst the remaining 33,800 stars save the two objects within the area covered by R325, and the catalogues do not add any clear case of variation: ratio of peculiar stars, 1 in 16,900.

It is interesting to consider that, on the basis of R325, in the hinterland beyond the reach of the V plates over the 40 square degrees covered, there are about 120,000 stars between $15^m{\cdot}0$ and $18^m{\cdot}5$, any one of which might have been brought within reach by a quite ordinary degree of variability.

The non-occurrence of any such case in this region is exactly in agreement with my invariable experience that an almost complete quiescence characterises the stars which border upon the confines of our stellar universe.

Tatsfield:
1914 *March* 20.

Since writing the above the following observations have been obtained:—

Plate.	Gr. Mean Time. 1914.	h	Limit.	Nova. m
V642	Mar. 19	9·3	$<g$	12·9
Da354	19	9·3	$<g$	12·85
R409	21	8·6	$<g$	12·75
V646	21	8·6	$<g$	12·85
Da358	21	8·6	$<g$	12·75
Da359	21	9·3	g	12·8
R415	22	9·0	$<g$	12·8
V655	22	9·0	$<g$	12·85
Da367	Mar. 22	9·0	$<g$	12·9
V660	27	10·3	$<g$	13·4
Da372	27	10·3	$<g$	13·3
R425	Apr. 2	10·4	$<g$	13·2
R431	4	9·7	$<g$	12·8
V681	4	9·7	g	12·9

Visually with the 15-inch, on 1914 April 4, $9^h{\cdot}1$, the Nova was slightly $>g$.

There is one early observation with this instrument, viz. 1911 October 26, Nova = *e*.

It appears that the decline in magnitude has not been accompanied by any marked change in colour, as the Nova is still decidedly brighter photographically than visually.

Plate R415, which was exposed for two hours, clearly records the double star *np* the Nova. Attention was also attracted to a companion to star 2 which had been overlooked in measuring the magnitudes of the faint stars near the Nova; it is situated about 6″ *sp* 2, in line between 2 and 5, magnitude about 19·3 from plates R255 and 325.

The superposition of R415 and 255 also drew attention to the existence of a faint nebula about 40′ S., the Nova at (1855) $3^h\ 21^m{\cdot}1$, N. 42° 45′. It is clearly recorded on four R plates, in form oval, major axis about 30″ lying *p* and *f*, nucleus central on the major and N. of centre on the minor axis: a star of photo-magnitude 12·4 lies about 1′ S.

Tatsfield:
1914 *April* 7.

Note on the Variable Star VV (17. 1912) *Aurigæ.* By H. G. Scott Barrett. (Plate 12.)

(*Communicated by Dr. A. A. Rambaut, M.A., F.R.S., Radcliffe Observer.*)

The variability of this star was detected as the result of an examination of the photographic plates taken with the 24-inch refractor for the determination of stellar parallax in the region of B.D. + 44°, 1408. It is unnecessary to enter here into the details of the evidence afforded by these plates, as all the material which they are capable of yielding has been included in the announcement of the discovery which appeared in the *Astronomische Nachrichten*, Nr. 4559. A further note was published in the *Astronomische Nachrichten*, Nr. 4620, giving the results of the observations as indicating the type of variation.

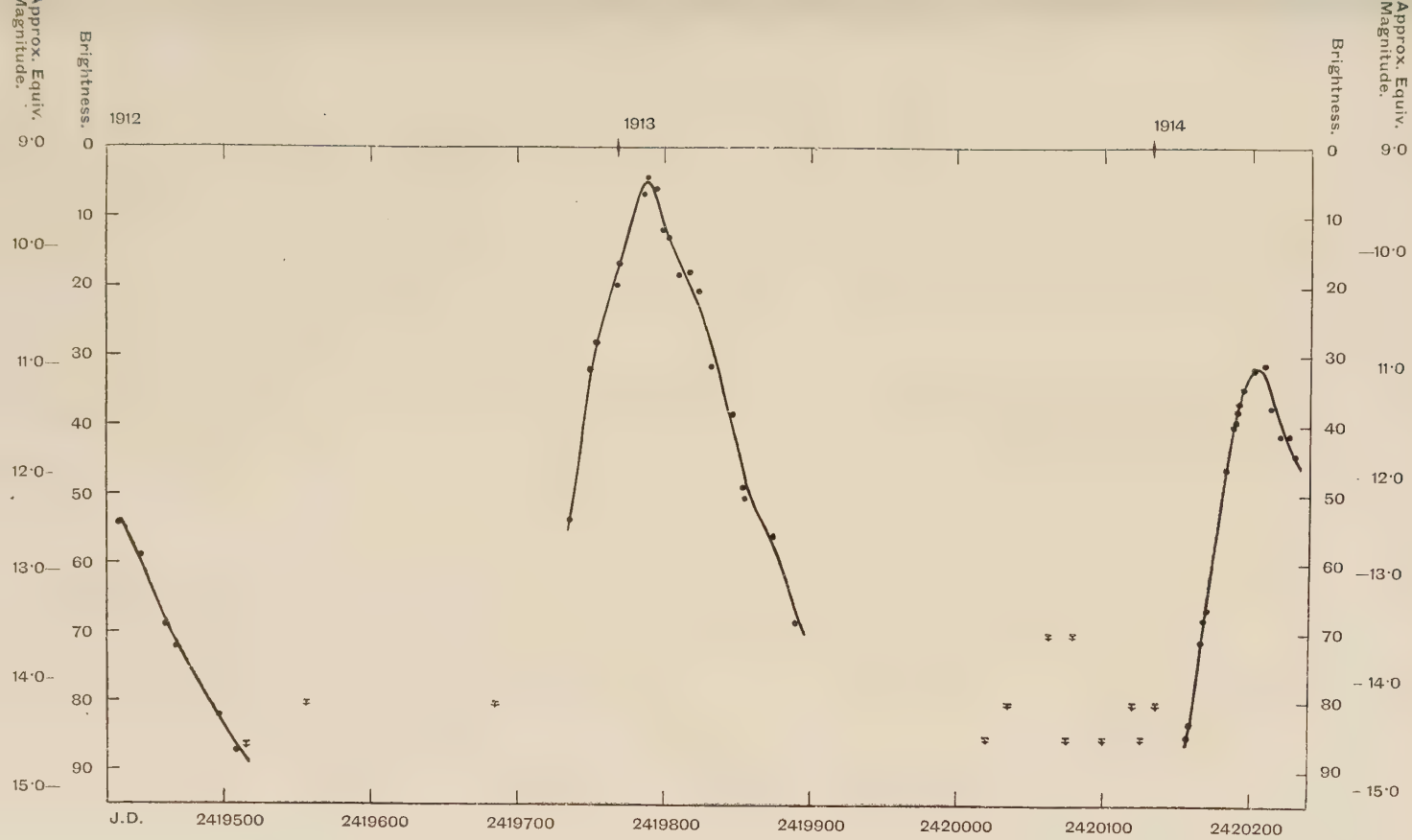
Since the discovery of the star, observations by Argelander's method have been made as frequently as circumstances would permit, mainly with the Barclay 10-inch refractor. The star, for the latitude of Oxford, is circumpolar, but it has been found to be very difficult, if not impossible, to secure observations near the lower culmination during the summer months, owing to the strong twilight. It may, however, be found possible to follow the star through a bright maximum when so placed.

A sequence of comparison stars extending from the ninth magnitude to the limit of visibility has been formed, but is at present incomplete as regards the faint stars. It would serve no useful purpose to give here the details of the light values attaching to the individual comparison stars, until sufficient observations have been made to give them a more or less definite character. In the meanwhile, it may be stated that the values in grades given in the second column of the table of observations below may be converted approximately into magnitudes by means of the relation:

$$\text{Magnitude} = 9{\cdot}0 + 0{\cdot}065n,$$

where n is the number of grades.

The following table of the results of the observations, extending from 1912 January 26 to 1914 April 6, contains in its respective columns the Julian date of observation, the brightness of the variable, and the number of separate comparisons entering into the determination of brightness. In the case of the few observations marked by an asterisk, the 18-inch telescope was employed.



Light Curve of VV (17, 1912) Aurigæ.

Date.	Brightness.	No. of Comparisons.
2419428	54	3
9443	59	4
9459	(69)	1
9468	(72)	1
9496	(82)*	1
9508	(87)*	3
9515	(<86)*	...
9556	(<80)*	Invisible.
9685	(<80)	Invisible.
9735	54	4
9749	32	5
9753	28	5
9767	20	2
9769	17	4
9786	7	2
9788	4	5
9794	6	4
9799	12	6
9803	13	4
9810	18	5
9817	18	4
9823	20	4
9832	31	4
9846	38	4
9854	49	4
9855	50	5
9865	53	2
9874	56	4
2419890	(69)	1
2420020	(<85)	Invisible.
0035	(<80)	Invisible.
0063	(<70)	Invisible.
0075	(<85)	Invisible.
0079	(<70)	Invisible
0100	(<85)	Invisible.
0120	(<80)	Invisible.
0126	(<85)	Invisible.
0136	(<80)	Invisible.
0157	(85)	1
0159	(83)	1
0166	(71)	1
0168	(68)	1
0169	67*	2
0182	46	5
0188	40	5
0189	40	5
0190	38	5
0191	37	4
0194	35	7
0202	32	5
0208	31	6
0213	37	8
0219	41	5
0225	41	3
0229	44	6

The values in brackets have been so treated by reason of the uncertainty of the scale for the faint comparison stars. No doubt attaches to the observations, but the values given are subject to modification when the sequence of comparison stars has been more accurately extended.

The observations have been plotted as shown on the accompanying diagram, and from the curve as drawn the star appears to have been at a maximum on J.D. 2419787 and J.D. 2420203. And as it is evident that a maximum could not have occurred between these dates, a period of 416 days is indicated.

From the photographic plates the star appears to have been bright on J.D. 2417265 and J.D. 2417275. Allowing for the difference of scale between the recent visual observations and the estimations published in the *Astronomische Nachrichten*, Nr. 4559, and for the fact, as observed whilst photographing the region on J.D. 2419450, that the photographic brightness is decidedly less than the visual, it may be assumed that the star was most probably within a month of its maximum at the epoch of the photographic observations—provided that, at the maximum in question, the star did not greatly exceed the brightness it attained on the occasion of the maximum of 1913.

Now, working backwards from the 1913 maximum and adopting a period of 416 days, a maximum is indicated, with an elapsed interval of six periods, at J.D. 2417291. This date appears, by comparison with the fact that the estimations show the star to be diminishing slightly in brightness between the two photographic observations, to be somewhat late. A rather longer period would then appear to fit these observations better. But from the conditions attending the storage, etc., of the parallax plates (which, moreover, in the present case were of a different make for each of the nights under consideration), this decrease in brightness is not altogether satisfactorily demonstrated. On the other hand, if we accept this decrease in brightness and assume the maximum to have occurred at about the first date, J.D. 2417265, we obtain a period of 420 days; but in this case, going back still further, we should have a maximum at about J.D. 2416845. Whilst this is not altogether an impossible date, it comes very dangerously near to being inconsistent with the observation of invisibility on J.D. 2416792. On the whole it appears that, whilst a slightly longer value of the period is possible, the following provisional elements may be adopted:

$$\text{Maximum} = 1913 \text{ January } 19 + 416^{d} \text{ E.}$$
$$= \text{J.D. } 2419787 + 416 \text{ E.}$$

An examination of the changes involved in the difference between the brightness attained by the star at the two maxima already obtained must be left for further observations.

Radcliffe Observatory, Oxford:
1914 *April* 7.

Occultation Results, 1912 and 1913. By Maurice E. J. Gheury and Nora E. Robertson.

At the beginning of 1912 two observations of occultations, made at Eltham and communicated to Dr. A. C. D. Crommelin, were stated by him to be of distinct value for correcting the Moon's place, being in good agreement with results obtained at the Greenwich Observatory. This knowledge, and the circumstance that, in clear weather, the time-ball of the Observatory (about 3 miles away) offers an excellent opportunity for chronometer rating, led to the undertaking of the observation of occultations as a routine work. The present paper gives the result of the first two years' work, and as it is intended to publish these results every year, it is trusted that some explanations as to the work itself will not be superfluous.

The first ten observations were made with a pocket chronometer of the stop-watch type, directly rated by means of the Greenwich time-ball. It was, however, found to possess a variable hourly temperature rate which prevented the attainment of an accuracy greater than 1·5 seconds.

The loan of a marine chronometer having been then secured, this was rated from the Greenwich time-ball by means of the stop-

watch started at the time-signal and stopped at a known chronometer time. The time of occultation was observed and referred to the chronometer exactly in the same manner. In this way displacements of the chronometer and its exposure to cold or damp are avoided (the observations being taken with a $3\frac{1}{2}$-inch Wray refractor taken out of doors on a garden stand), the hourly rate of the watch is entirely eliminated, as the "run" of the stop-watch rarely exceeds half a minute, and there is no risk of "slipping" seconds. Special tests were made from time to time to ascertain that the stop-watch mechanism worked in a perfect manner. The observer trained himself to estimate his personal error or "lag" between the signal or occultation and the "click" indicating the starting of the stop-watch mechanism.

One observation consisted of two time-signal observations, one before and one after the occultation, at twenty-four hours' (or sometimes forty-eight hours') interval, and of the observation of the occultation itself. Each time determination is affected by a possible error of $\pm 0^{s}\cdot 2$ second ($\pm 0^{s}\cdot 1$ second in estimating the "lag," and $\pm 0^{s}\cdot 1$ second in the comparison of the stop-watch and the chronometer), and repeated tests showed that the error was not likely to be more considerable. The accuracy obtained is then approximately $0^{s}\cdot 5$ second, on the assumption that the rate of the chronometer is practically constant between the two observations of time-signals. In some cases, the weather being misty, the signal had to be obtained from a spot near the Observatory, and a correction for the hourly rate of the watch during the journey home applied, with a corresponding loss of accuracy.

The times given in the table are the actual times at which the occultations took place, as deduced from the observations, all the corrections having been made—an accuracy of $0^{s}\cdot 1$ second being aimed at. The time-signals in 1912 were assumed to be correct, as the reductions were completed before a knowledge of the errors was obtained. The correction to the given times, calculated from the final clock errors, are given in the notes following Table I. whenever it is as much as $0^{s}\cdot 1$ second. In 1913 the observed times were corrected before reduction.

The places of observation vary according to position and altitude of star: they have been designated by letters A, B, C, D, E, and F (Table I., column 9), and are as follows:—

Position.	Astronomical Latitude N.	Geocentric Latitude N.	Longitude E.	Height above Mean Sea-level.
	° ′ ″	° ′ ″	h m s	feet.
A	51 27 13·5	51 15 54·9	0 0 15·13	246
B	13·6	55·0	15·14	260
C	13·7	55·1	15·16	260
D	13·7	55·1	15·17	248
E	13·8	55·2	15·20	248
F	13·9	55·3	15·23	248
G	13·6	55·0	15·09	244

The occultations the number of which is distinguished by a * have not their particulars given in the *Nautical Almanac*. For

these stars the values of $\Delta\alpha$, $\Delta\delta$ have been, in 1912, found by interpolation from the values given in the *Nautical Almanac* for the neighbouring stars; in 1913 the values of $\Delta\alpha$, $\Delta\delta$ were calculated from Bessel's *Day Numbers* in the usual manner. The positions of these stars and of a few others between 6·5 and 7·0 magnitude, which are not given in the *Nautical Almanac*, are given below:—

No.	Year.	Mean R.A.	Mean N.P.D.	Catalogue.
		h m s	° ′ ″	
4*	1912·0	5 50 8·20	62 17 59·3	*Astr. Gesel.*, 1875.
5*	,,	5 53 33·89	62 26 45·2	,,
10*	,,	7 57 37·49	64 46 11·2	,,
11	,,	13 53 45·19	103 41 36·4	*Wash. Zod. Stars*, 1920.
16	1913·0	23 24 17·64	95 52 5·15	*Vienna Ottakring*, 1893.
20	,,	10 22 35·52	78 14 24·07	*Edin. Zod. Stars*, 1900.
23*	,,	7 7 8·24	63 17 20·8	*Astr. Gesel.*, 1875.
25*	,,	7 11 53·78	63 28 53·1	,,
26*	,,	7 12 32·72	63 30 38·8	,,

For the other stars the R.A. and N.P.D. are those given in the *Nautical Almanac*. The angle ϕ (Table I., column 8) is the angle between the north direction and the normal to the direction of the Moon's motion.

To ensure complete security against any clerical error, the reduction of the observations was made independently by both the authors. The method followed is that in use at the Greenwich Observatory (Introduction to Greenwich Observations, 1843–1849). The Parallax, Right Ascension, and North Polar Distance of the Moon were computed from second differences. The North Polar Distance of the corresponding point was calculated by three successive approximations. The semidiameter was obtained on the basis of Struve's figure 15′ 32″·65, and the value of the compression used in the geocentric reduction was $\frac{1}{297}$, both these values being now used at Greenwich.

Whenever an occultation was observed simultaneously at Greenwich and Eltham, the Greenwich value of the semidiameter − dist. is given for comparison (Table II.).

Assuming—

True time of phenomenon = Observed time $+ t$ sec.

,, R.A. of Moon = *Naut. Alm.* value $+ x'' + t \times$ motn. in 1 sec.

,, N.P.D. of Moon = ,, ,, $+ y'' + t \times$ motn. in 1 sec.

,, R.A. of star = ,, ,, $+ e''$

,, N.P.D. of star = ,, ,, $+ f''$

,, Parallax of Moon = ,, ,, $\times \left(1 + \frac{m}{1000}\right)$

Semidiameter of moon = ,, Parallax $\times$ ·272535$\left(1 + \frac{n}{1000}\right)$,

the equations of condition given in Table II. have been found.

The best thanks of the authors are due to Dr. A. C. D. Crommelin for his patient and indefatigable help, and to Mr. T. Lewis for his readiness to supply the necessary information as to time-signals.

TABLE I.

No.	Date.	Time.	Accuracy.	Star.	Mag.	Dis. or Reap.	ϕ.	Position.
	1912	h m s	s				°	
1	Jan. 27	11 43 2·7	±1·5	π Arietis	5·2	D	72	B
2	28	13 45 55·1	±1·5	B.A.C. 1189	5·9	D	50	B
3	Apr. 21	7 49 54·5	±1·1	136 Tauri	4·6	D	62	B
4*	21	8 45 24·9	±1·2	W.B. (2) V 1531	7·5	D	88	B
5*	21	10 5 16·4	±1·4	W.B. (2) V 1644	7·9	D	69	B
6	21	10 45 51·1	±1·5	B.A.C. 1918	6·1	D	85	B
7	22	12 53 49·7	±1·2	47 Geminorum	5·6	D	85	G
8	23	7 17 32·8	±1·0	ω_1 Cancri	6·1	D	84	D
9	23	8 4 48·8	±1·5	ω_2 Cancri	6·2	D	32	D
10*	23	8 41 20·3	±1·6	W.B. (2) VII 1503	8·5	D	31	D
11	May 27	11 15 46·0	±0·5	B.D. −13° 3802	6·8	D	69	F
12	June 20	10 44 4·3	±0·5	σ Leonis	4·2	D	88	B
13	Sept. 22	11 48 13·8	±0·5	κ Capricorni	4·8	D	81	B
14	Dec. 17	6 0 45·2	±0·5	44 Piscium	6·0	D	8	D
15	20	6 0 32·3	±0·5	40 Arietis	6·0	D	24	D
	1913							
16	Jan. 12	6 1 36·0	±0·9	B.D. −6° 6220	7·0	D	65	B
17	18	9 3 46·6	±0·5	χ Tauri	5·3	D	84	D
18	20	10 36 42·8	±0·9	49 Aurigæ	5·1	D	57	D
19	Mar. 23	9 22 17·8	±0·5	α Virginis	1·2	R	36	D
20	Apr. 16	12 23 23·6	±0·5	B.D. +12° 2211	6·6	D	16	A
21	22	15 39 37·9	±·07	π Scorpii	3·1	R	84	B
22	May 10	7 48 36·7	±·08	47 Geminorum	5·6	D	84	B
23*	10	8 35 32·0	±0·6	B.D. +26° 1487	7·8	D	35	B
24	10	10 15 8·8	±0·5	B.A.C. 2383	6·5	D	48	B
25*	10	10 21 18·0	±0·6	B.D. +26° 1510	7·4	D	60	B
26*	10	10 35 33·6	±0·5	B.D. +26° 1512	7·9	D	64	B
27	13	11 19 31·5	±0·6	34 Leonis	6·4	D	65	F
28	June 17	10 15 26·4	±1·0	B.A.C. 5737	6·7	D	73	F
29	Dec. 31	5 44 10·4	±0·5	ι Aquarii	4·4	D	24	B

Notes.—Nos. 2, 5*: the star seemed to vacillate at disappearance.
No. 19: not quite instantaneous.
No. 21: reappearance peculiar, see *Observatory*, 1913 June, p. 268.
No. 23*: became dim near the limb.
No. 24: disappearance gradual.
Corrections to Time of Phenomenon, from final clock errors:—
Nos. 1 and 11, −0ˢ·2; Nos. 2 and 15, −0ˢ·1; Nos. 7, 8, 9, and 14, +0ˢ·1; Nos. 12 and 13, +0ˢ·2.

TABLE II.

No.	Semi-diameter – Dist. From Obs.	Semi-diameter – Dist. at Greenwich Observatory.	$= \frac{e}{-x}\} \times$.	$+f\times$.	$+y\times$.	$+t\times$.	$+m\times$.	$+n\times$.
1	– 12·92		+0·950	–0·060	+0·062	–0·511	+2·298	–0·952
2	– 10·55		+0·845	+0·402	–0·400	–0·489	+0·942	–0·967
3	– 10·91	– 9·85	+0·798	+0·425	–0·423	–0·477	+0·920	–0·969
4*	– 14·08		+0·878	–0·082	+0·085	–0·541	+2·299	–0·970
5*	– 11·55	– 10·67	+0·855	+0·240	–0·237	–0·593	+1·557	–0·970
6	– 14·68		+0·878	+0·072	–0·070	–0·627	+1·972	–0·970
7	– 12·32		+0 884	+0·013	–0·011	–0·664	+1·929	–0·971
8	– 13·08		+0·865	+0·270	–0·268	–0·456	+0·368	–0·970
9	– 7·89	– 7·04	+0·331	+0·930	–0·929	–0·312	–1·137	–0·970
10*	– 5·48		+0·307	+0·940	–0·940	–0·315	–1·186	–0·970
11	– 11·06		+0·685	+0·711	–0·712	–0·404	–1·505	–0·927
12	– 11·43	– 11·24	+0·893	+0·435	–0·435	–0·567	+0·789	–0·963
13	– 11·41		+0·924	–0·228	+0·226	–0·380	+1·779	–0·886
14	– 0·35	– 0·04	–0·336	–0·942	+0·942	–0·127	+2·485	–0·902
15	– 6·11		+0·678	+0·699	–0·698	–0·122	–2·463	–0·939
16	– 10·10	– 10·93	+0·594	–0·803	+0·803	–0·379	+2·789	–0·886
17	– 13·81	– 13·94	+0·893	–0·127	+0·129	–0·419	+0·547	–0·955
18	– 10·46		+0·752	–0·517	+0·519	–0·370	+0·785	–0·986
19	+ 6·23	+ 6·54	–0·133	–0·991	+0·991	+0·364	+3·217	–0·985
20	– 0·71		–0·173	+0·984	–0·984	–0·207	–2·768	–0·983
21	+ 11·02		–0·901	–0·107	+0·105	+0·426	–0·637	–0·952
22	– 12·45		+0·864	+0·227	–0·224	–0·505	+1·386	–0·953
23*	– 6·55		+0·412	+0·886	–0·886	–0·368	–0·904	–0·953
24	– 10·44	– 10·49	+0·741	–0·549	+0·551	–0·397	+3·192	–0·954
25*	– 13·78		+0·705	+0·609	–0·608	–0·555	+0·185	–0·954
26*	– 12·46		+0·722	+0·584	–0·582	–0·570	+0·244	–0·954
27	– 14·63	– 13·88	+0·968	+0·010	–0·008	–0·483	+2·055	–0·970
28	– 12·65	– 12·18	+0·876	–0·155	+0·153	–0·383	–0·041	–0·933
29	– 4·14	– 4·49	–0·014	–1·000	+1·000	–0·232	+2·889	–0·900

Woolwich Polytechnic:
1914 *March.*

Note on the Method of Reduction of the Paris Wireless Rhythmic Signals. By R. A. Sampson, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland. (Plate 13.)

The purpose of the following note is to draw attention to the method of reduction of the Paris wireless rhythmic time signals, as given in the publication *Réception des Signaux Radiotélégraphiques*, 2nd edition, 1913, p. 65. It appears that this method hardly utilises all the information which the observation of the coincidences supplies, and that this might be remedied with very slight additional work. Taking the example given as an illustration, the numbers observed are as below :—

Coincidence.	Break.	Number of Signal 1.
h m s	m s	
10 7 31		[51]
	7 40	60
8 21		[102]
	8 39	120
9 10		[152]
	9 38	180
10 0		[203]
	10 37	240
10 50		[254]*
	11 36	300

* By a misprint this is given as 264 in the model, p. 65.

The numbers in the third column corresponding to the breaks, or last signal, are, of course, data. The numbers in [ ] corresponding to the coincidences are deduced as explained in the pamphlet. The method of reduction is then to take the mean interval between coincidences,

$$\tfrac{1}{4}[50^s + 49^s + 50^s + 50^s] = 49^s{\cdot}75,$$

and, increasing this by a unit, enter the tables given with interval $1 - 1/50{\cdot}75$, and extrapolate for the first and last signals by applying to the first and last observed coincidences the entries for 50 and 46 intervals respectively, obtaining

$$10^h\ 7^m\ 31^s - 49^s{\cdot}015 = 10^h\ 6^m\ 41^s{\cdot}985$$

and

$$10\ 10\ 50 + 45{\cdot}093 = 10\ 11\ 35{\cdot}093$$

as the moments when the first and the three-hundredth signal respectively were sent out.

It will be noticed that the only use which is made of the second, third, and fourth coincidences is to obtain the intervals, and finally, when the intervals are meaned, the observations of these coincidences are eliminated completely. The method which I propose, and which we use at Edinburgh, is to obtain the intervals as above, and then extrapolate for the first and the three-hundredth signal from each observation of coincidence separately.

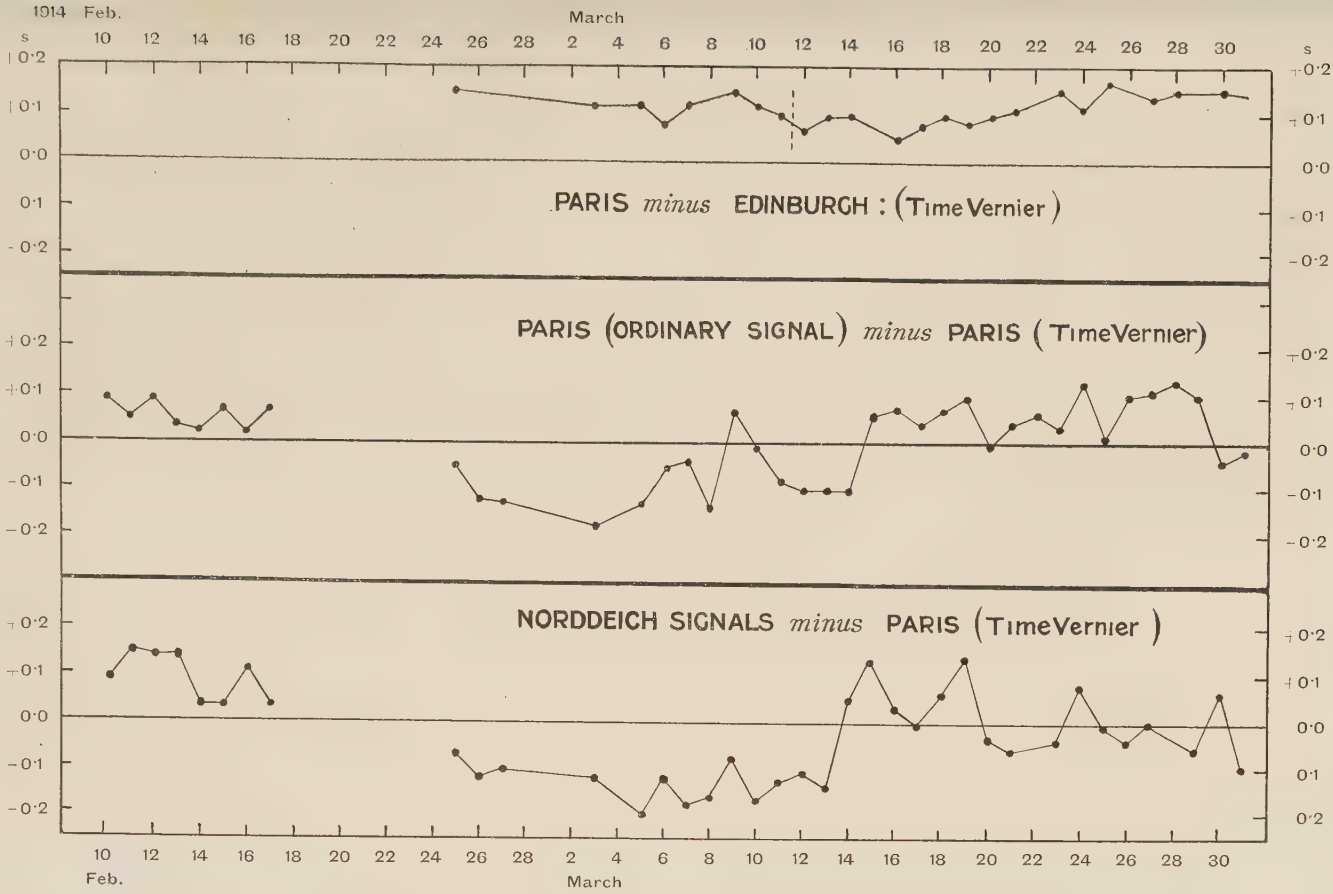
It will result from the method of deriving the interval that the

moment of the first signal calculated from the last coincidence, and that of the last signal calculated from the first coincidence, will agree. The remaining three are completely independent, and appear to me a valuable proof of the observation, offering material that will occasionally lead one to adopt a correction of one or two hundredths of a second to the times indicated from the first and last coincidences alone. The sum is very quickly done with the aid of the published tables, and in the example referred to above the necessary figures would run :—

Coincidence.	Number of Signal.	Corrn.	First Signal.	Corrn.	Last Signal.
h m s			m s		m s
10 7 31	51	(−50)		(+249)	
		−49ˢ·02	6 41·98	+4ᵐ 4ˢ·09	11 35·09
8 21	102	(−101)		(+198)	
		−1ᵐ 39ˢ·01	41·99	+3ᵐ 14ˢ·09	35·09
9 10	152	(−151)		(+148)	
		−2ᵐ 28ˢ·02	41·98	2ᵐ 25ˢ·08	35·08
10 0	203	(−202)		(+97)	
		−3ᵐ 18ˢ·02	41·98	+1ᵐ 35ˢ·09	35·09
10 50	254	(−253)		(+46)	
		−4ᵐ 8ˢ·01	41·99	+45ˢ·09	35·09
		Adopt	6 41·98		11 35·09

It will not escape remark that this close confirmation arises from the approximate equality of the observed intervals between coincidences, and that upon occasions when all four intervals are equal, every figure is in agreement with every other; so that a test more or less equivalent to the above can be obtained without actually reducing the second, third, or fourth coincidences, by a mere inspection of the intervals. But in my opinion the reduction is worth making.

As not very much has hitherto been published regarding results obtained at different observatories by reception of wireless time-signals, it may be of interest to record the figures from Edinburgh during the past month. It may be explained that the error of the standard mean-time clock Riefler is obtained by comparing its prick upon the tape of the Fuss chronograph with that of the standard sidereal clock; the stroke of the chronograph is then the standard of time by Riefler. Riefler drives a controlled clock, which carries low down on its pendulum a very small wheel which makes a very brief contact by striking upwards an agate knife-edge at each beat. Wires are led to the observing table and round the primary of an induction coil. The primary slides within the secondary, and the terminals of the secondary are led to the telephone. By this means the loudness of the tick can be controlled, and its length can be controlled by regulating the contact. The resistance of the secondary must be much greater than that of the telephone, or the external signals will be weakened. Between this tick in the telephone and the time of stroke of the chronograph there is a definite lag. This is measured by mounting


1914 Feb.
March
s
10 12 14 16 18 20 22 24 26 28 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
+0·2
+0·1
0·0
-0·1
-0·2
PARIS minus EDINBURGH : (Time Vernier)
PARIS (ORDINARY SIGNAL) minus PARIS (Time Vernier)
NORDDEICH SIGNALS minus PARIS (Time Vernier)
Feb.
March

close to the observing table and to the chronograph a free pendulum, which makes a tick at each oscillation and is regulated so as to gain about one beat in about fifty-five upon mean-time seconds. The coincidences of this with the controlled tick in the telephone and the stroke of the chronograph can be taken in a satisfactory way for several minutes in succession. The lag is determined each night by these means, and appears to vary slightly between $+^{s}\cdot10$ and $+^{s}\cdot15$, according to the state of regulation of the contact. It is at present determined about half-past ten each night, and applied at the value found to the following set of observations. In this series there is a break at March 12, as the contact wheel was changed at this date; the correction for lag was not satisfactorily determined until March 20, and between March 12 and 20 it is applied at the constant value $+^{s}\cdot11$.

Observations of Clock Correction Paris–Edinburgh by Paris Rhythmic Signals.

Date.	Obsr.	Note.	Obsd. P.–Ed.	Riefler's Error.	Corrd. P.–Ed.	Chron.-Tick Lag.	Adopted P.–Ed.
Mar. 3	J.S.	...	+·34	+·22	+·12		
5	J.S.	v. good sigs.	+·40	+·28	+·12		
6	R.A.S.	...	+·36	+·28	+·12		
ib.	J.S.	...	*ib.*	*ib.*	*ib.*		
7	J.S.	...	+·40	+·28	+·12		
9	J.S.	...	+·35	+·20	+·15		
10	R.A.S.	v. good sigs.	+·36	+·24	+·12		
11	J.S.	...	+·42	+·32	+·10		
		New contact wheel introduced.					
12	R.A.S.	...	+·42	+·24	+·18	[+·11]	+·07
13	J.S.	several sigs. missing	+·46	+·25	+·21	*ib.*	+·10
14	R.A.S.	...	+·46	+·25	+·21	*ib.*	+·10
16	J.S.	some sigs. faint	+·46	+·30	+·16	*ib.*	+·05
17	R.A.S.	...	+·45	+·26	+·19	*ib.*	+·08
ib.	J.S.	...	*ib.*	*ib.*	*ib.*	*ib.*	*ib.*
18	J.S.	...	+·49	+·28	+·21	*ib.*	+·10
19	J.S.	...	+·47	+·28	+·19	*ib.*	+·08
20	R.A.S.	...	+·53	+·32	+·21	+·11	+·10
21	J.S.	sigs. faint, atmos.	+·56	+·35	+·21	+·10	+·11
23	J.S.	...	+·62	+·37	+·25	+·10	+·15
24	R.A.S.	poor obsn.	[+·61]	+·38	+·23	+·12	[+·11]
25	J.S.	*	+·68, +·76	+·39	?+·29	+·12	?+·17
27	J.S.	*	+·54, +·63	+·30	?+·24	+·11	?+·13
28	R.A.S.	...	+·49	+23	+·26	+·11	+·15
30	R.A.S.	...	+·38	+·10	+·28	+·13	+·15
31	J.S.	...	+·34	+·06	+·28	+·14	+·14

* On these two days the determinations of first and last signals were accordant among themselves but discordant with the Paris determinations.

These results must be called remarkably smooth, considering

that, besides any error proper to the observation, they contain the whole of any accidental error there may be in determination of the clock corrections of the two observatories. They appear also to be quite free from personality. It seems clear that the Paris rhythmic signals have created one of the most delicate and reliable resources of astronomy.

The ordinary signals sent out from Paris and Norddeich are naturally upon a different plane; but their summary presents some features of interest. In the following table, in order to clear the results from the changes of system due to our experiments with different methods of producing a tick in the telephone, the numbers given are referred to the time of Paris as given by its rhythmic signals, by subtracting *P.–Ed.* as shown above, from *P.–Ed.* or *ND.–Ed.* as determined from the ordinary signals. These latter are the mean of all observations by both observers on each day. No distinction has been made between the Paris 10 A.M. signals, and those of $10\frac{3}{4}$ A.M. and $11\frac{3}{4}$ P.M., though these latter are relatively of much smaller weight.

Comparison of Time of Paris Ordinary Signals and Norddeich Signals with Time by Paris Rhythmic Signals.

Date.	Paris.	Norddeich.	Date.	Paris.	Norddeich.
	s	s		s	s
Feb. 10	+·09	+·09	Mar. 12	−·10	−·11
11	·05	·15	13	·10	−·14
12	·09	·14	14	−·10	+·05
13	·03	·14	15	+·06	·13
14	·02	·03	16	·07	·03
15	·07	·03	17	·04	·00
16	·02	·11	18	·07	·06
17	+·07	+·04	19	+·10	+·14
. . . . .			20	·00	−·04
25	−·05	−·07	21	+·04	−·06
26	·12	·12	22	·06	..
27	−·13	−·10	23	·03	−·04
. . . . .			24	·13	+·08
Mar. 3	−·18	−·12	25	·01	−·01
5	·13	·20	26	·10	−·04
6	·05	·12	27	·11	·00
7	·04	·18	28	·13	−·39
8	−·14	·16	29	+·10	−·06
9	+·07	·08	30	−·04	+·07
10	−·01	·17	31	−·02	−·10
11	−·08	−·13			

Apart from the Norddeich observation of March 28, the whole are comprised between $\pm 0^{s}\cdot 2$. Omitting this observation, the greatest discrepancy between successive days is $0^{s}\cdot 21$, and the average discrepancy for either series $0^{s}\cdot 06$.

Erratum in Annual Report, vol. lxxiv. p. 341, *first line following the Table:*

for A.J. No. 229 *read* L.O.B. No. 229.